The relationship between the required return on debt and equity

Report for DBP NGP Pty Ltd

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1. Background and conclusions

Overview

1. SFG Consulting (**SFG**) has been retained by DBP NGP Pty Ltd (**DBP**) to consider frameworks for evaluating the consistency between the required return on debt and the required return on equity in the Australian regulatory setting.

2. In particular, the National Gas Rules set out the **allowed rate of return objective**:

> [t]he rate of return for a [Service Provider] is to be commensurate the efficient financing costs of a benchmark efficient entity with a similar degree of risk as that which applies to the [Service Provider] in respect of the provision of [services].[1]

3. The National Gas Rules also provide guidance on the implementation of the allowed rate of return objective. The Rules refer to the desirability of using an approach that leads to the consistent application of financial parameters that are common to the return on equity and the return on debt,[2] and to consideration of any interrelationships between estimates of financial parameters that are relevant to the estimates of the return on equity and the return on debt.[3]

4. There is an interrelationship between the return on equity and the return on debt because both equity and debt securities depend on the assets of the same firm. Debt and equity simply represent different claims over the *same* assets. Consequently, there is an interrelationship between the return on equity and the return on debt, the estimate of one is relevant to the estimate of the other, and the two estimates must be consistent with each other.

5. That is, the linkage between the required returns on debt and equity in the same benchmark firm appears to be central to the NGR 87(5) requirements to have regard to all relevant evidence, consistency, and interrelationships between parameters for equity and debt.

6. In this report, we consider the standard framework for modelling the linkage between the required returns on debt and equity in the same firm. We explain the basis for the standard modelling framework, the implications for the relationship between the return on equity and the return on debt in the same firm, and we summarise the relevant literature on the development and application of this modelling framework.

Preparation of this report

7. This report has been authored by Professor Stephen Gray, Professor of Finance at the UQ Business School, University of Queensland and Director of SFG Consulting, a specialist corporate finance consultancy. I have Honours degrees in Commerce and Law from the University of Queensland and a PhD in financial economics from Stanford University. I teach graduate level courses with a focus on cost of capital issues, I have published widely in high-level academic journals, and I have more than 15 years' experience advising regulators, government agencies and regulated businesses on cost of capital issues.

8. My opinions set out in this report are based on the specialist knowledge acquired from my training and experience set out above.

[1] For example, see NGR 87(2)(3).
[2] National Gas Rules, clause 87(5)(b).
[3] National Gas Rules, clause 87(5)(c).

9. I have read, understood and complied with the Federal Court of Australia Practice Note CM7 *Expert Witnesses in Proceedings in the Federal Court of Australia*.

10. My instructions are attached as an appendix to this report.

Summary of conclusions

11. The main conclusions from this report are as follows.

12. Merton (1974) noted the simple fact that equity and debt are contingent claims over the assets of the same firm. Both become less valuable as the assets of the firm decline in value and both become more valuable as the assets of the firm rise in value. Both are linked to the value of the assets of the firm. Thus, if there are certain factors that drive changes in the value of the assets of the firm, those same factors will drive the returns to debt and equity in that firm. This means that there is a positive relationship between the return on debt and the return on equity in the same firm.

13. The original framework developed by Merton (1974) has spawned a huge literature that has developed and refined the framework in a number of directions. In all cases, the strong link between the required returns on debt and equity remains a central feature.

14. Current empirical applications of the Merton framework generate a number of important insights that are relevant to the regulatory setting:

    a) There is a positive relationship between the expected return on equity and the expected return on debt;

    b) The expected return on debt is equal to the yield on debt, but for the chance of default. Consequently, the expected return on debt is closely approximated by the yield when the probability of default is low;

    c) If a regulator considers that their regulatory allowance will be sufficient to ensure the solvency of the regulated firm, there must be a positive relationship between the allowed return on equity and the allowed return on debt. It would be inconsistent for a regulator to materially increase the allowed risk premium on debt, but to make no change to the allowed risk premium on equity. The evolution of the allowed risk premium on debt is relevant evidence to consider when determining the risk premium to be allowed on equity;

    d) A high DRP (yield less risk-free rate) need not imply a high default probability – where defaults are systematically more likely to occur during recessions and financial crises (which they are); and

    e) If the regulator considers that there *is* a material chance of default, the allowed return on equity would need to be grossed-up (as explained above) to ensure that the *expected* return to equity is consistent with the regulator's estimate of the required return.

2. Relevant regulatory setting

Overview

15. The National Gas Rules require that the allowed rate of return must achieve the **allowed rate of return objective**:

> [t]he rate of return for a [Service Provider] is to be commensurate the efficient financing costs of a benchmark efficient entity with a similar degree of risk as that which applies to the [Service Provider] in respect of the provision of [services].[4]

16. Section 87(5) of the National Gas Rules provides some guidance on the implementation of the allowed rate of return objective:

> In determining the *allowed rate of return*, regard must be had to:
>
> (a) relevant estimation methods, financial models, market data and other evidence;
>
> (b) the desirability of using an approach that leads to the consistent application of any estimates of financial parameters that are relevant to the estimates of, and that are common to, the return on equity and the return on debt; and
>
> (c) any interrelationships between estimates of financial parameters that are relevant to the estimates of the return on equity and the return on debt.[5]

17. The focus of this report is on the relationship between the allowed return on debt and the allowed return on equity under the requirements of NGR 87(5). The theme of 87(5) is that the regulator should properly consider all relevant evidence in producing an allowed return that has been computed in an internally consistent manner.

Consistency NGR 87(5)(b)

18. We begin by considering the 87(5)(b) consistency requirement. We note that the regulator must produce estimates of the required return on equity and the required return on debt as two of the parameters in the WACC formula:

$$WACC = r_e \frac{E}{V} + r_d \frac{D}{V}.$$

19. It is possible to consider whether there has been a consistent application of the estimates of these two financial parameters at several levels. For example, the Guideline indicates that the ERA will ensure that its estimate of the required return on equity is greater than its estimate of the required return on debt:

> The Authority considers that in general, the return on equity should exceed the return on debt, given that equity is more risky than debt. [6]

and that:

[4] For example, see NGR 87(2)(3).
[5] National Gas Rules, clause 87(5).
[6] ERA Rate of Return Guideline, Appendix 29, p. 204, Paragraph 70.

> prevailing market conditions should also be taken into account when determining whether the relativities between the return on debt and equity are reasonable at the time the regulatory decisions are made. [7]

20. We agree that such a comparison is one of the *necessary* preconditions for consistency – given that equity in the benchmark firm must be riskier than debt in the same benchmark firm at the same point in time, it must be the case that the required return on equity is higher than the required return on debt. However, this is not a *sufficient* condition for consistency – it is possible that the estimates of the required returns on equity and debt are inconsistent even though the return on equity is higher than the return on debt.

21. The approach that we adopt throughout this report is to use all relevant evidence to determine whether the return on equity and the return on debt have been estimated in a consistent manner. Confirming that the required return on equity is higher than the required return on debt is one check for consistency, but it is not the only relevant evidence of consistency.

Interrelationships NGR 87(5)(c)

22. NGR 87(5)(c) requires the regulator to have regard to any interrelationships between estimates of financial parameters that are relevant to the estimates of the return on equity and the return on debt. In the regulatory setting the return on equity and the return on debt are both estimated by adding a risk premium to the risk-free rate. The return on equity is estimated as the risk-free rate plus a premium to reflect the risk of owning equity in the benchmark firm. The return on debt is estimated as the risk-free rate plus a premium to reflect the risk of owning debt in the benchmark firm. There is a clear interrelationship between the equity risk premium for the benchmark firm and the debt risk premium for the same benchmark firm at the same point in time. The remainder of this report illustrates the structure of that relationship and shows how a regulator would have regard to that interrelationship under NGR 87(5).

Relevant evidence NGR 87(5)(a)

23. NGR 87(5)(a) requires the regulator to have regard to all relevant evidence. In our view, information about the debt risk premium is relevant evidence when estimating the equity risk premium for the same benchmark firm. For example, knowing that the debt risk premium for the benchmark firm has tripled over the previous two years would be relevant evidence when estimating the equity risk premium for the same benchmark firm.

Balance of report

24. In our view, it would be inconsistent for a regulator to materially increase the debt risk premium over the same period that it reduced the equity risk premium for the same firm. The reason that there must be a consistency between the debt risk premium and the equity risk premium for the same firm is that both depend on the assets of the same firm. Debt and equity simply represent different claims over the *same* assets. This linkage between the required returns on debt and equity in the same benchmark firm appears to be central to the NGR 87(5) requirements to have regard to all relevant evidence, consistency, and interrelationships between parameters for equity and debt. In this report, we consider the standard framework for modelling the linkage between the required returns on debt and equity in the same firm.

[7] ERA Rate of Return Guideline, Appendix 29, p. 204, Paragraph 72.

3. Contingent claims framework

Overview

25. In this section of the report, we set out the framework that has been developed in the finance literature for deriving the relationship between the required returns on debt and equity in the same firm. This literature dates back to the contingent claims framework that was originally developed by Merton (1974) as part of his Nobel Prize winning portfolio of work. Merton noted the simple fact that equity and debt are contingent claims over the assets of the same firm. Both become less valuable as the assets of the firm decline in value and both become more valuable as the assets of the firm rise in value. Both are linked to the value of the assets of the firm. Thus, if there are certain factors that drive changes in the value of the assets of the firm, those same factors will drive the returns to debt and equity in that firm. Consequently, there must be a positive relationship between the return on debt and the return on equity in the same firm.

26. A number of recent papers published in leading finance journals have examined the relationship between the expected returns on debt and equity in the same firm within the standard Merton framework. For example, Campello, Chen and Zhang (2008) state that:

> Motivated by Merton (1974), our basic approach recognizes that debt and equity are contingent claims written on the same productive assets and thus must share similar common risk factors. The upshot of this observation is that we can use corporate bond data to glean additional information about investors' required equity rates of returns. [8]

and that:

> Our basic idea is that bond and equity risk premiums are intrinsically linked because equity and bonds are contingent claims written on the same productive assets, an insight that can be traced back to Merton (1974). [9]

The Merton model

27. Merton noted the simple fact that equity and debt are contingent claims over the assets of the same firm. Both become less valuable as the assets of the firm decline in value and both become more valuable as the assets of the firm rise in value. Both are linked to the value of the assets of the firm. Thus, if there are certain factors that drive changes in the value of the assets of the firm, those same factors will drive the returns to debt and equity in that firm. This means that there is a positive relationship between the return on debt and the return on equity in the same firm.

Equity as a call option on the value of the firm

28. A **call option** is a financial security that provides the holder with the right, but not the obligation to buy a particular asset at a particular price on a particular future date. The holder is free to exercise the option (by buying the asset at the specified price) or to let the option lapse. The price paid when exercising the option is known as the *strike price*. For example, consider a $5 call option on Telstra shares that matures at the end of March. If the Telstra share price is $6 at the end of March, the holder will exercise the option, which involves paying $5 for a Telstra share, which is worth $6 at the time. In this case, the holder of the call option makes a gain of $1 by exercising, and whoever sold them that call option makes a loss of $1 when the option is exercised against them. If the Telstra

[8] Campello, Chen and Zhang (2008), p. 1298.
[9] Campello, Chen and Zhang (2008), p. 1298.

share price is $4.50 at the end of March, the holder will let the option lapse – they will not want to pay $5 for an asset that is worth only $4.50.

29. Merton (1974) models the equity of a firm as a call option on the firm's assets, with a strike price equal to the face value of the firm's debt. In the Merton model, default can occur only at debt maturity.

30. To see how this insight works, consider the following simple example. Suppose that a firm has a single source of debt funding with repayment of $5 million due in one year. Consider two cases at the time the repayment is due:

    a) Scenario 1: The assets of the firm are worth $9 million; and

    b) Scenario 2: The assets of the firm are worth $3 million

31. In Scenario 1 the lenders will receive the full $5 million repayment that they were promised and the equity holders receive the residual $4 million ($9 million - $5 million).

32. In Scenario 2 the firm defaults on its debt, the lenders take over the assets and receive $3 million and the equity holders receive nothing (though due to limited liability they have no further obligations).

33. In this case, the payment to the equity holders is identical to the payoff of a call option written on the assets of the firm with a strike price equal to the face value of the debt (in this case $5 million).

Debt as short a put option on the value of the firm

34. A **put option** is a financial security that provides the holder with the right, but not the obligation to sell a particular asset at a particular price on a particular future date. The holder is free to exercise the option (by selling the asset at the specified price) or to let the option lapse. The price received when exercising the option is known as the *strike price*. For example, consider a $5 put option on Telstra shares that matures at the end of March. If the Telstra share price is $6 at the end of March, the holder will allow the option to lapse – the holder will not want to sell a Telstra share for $5 if it is worth $6. If the Telstra share price is $4.50 at the end of March, the holder will exercise the put option. This involves selling a Telstra share for $5 when it is worth only $4.50 at the time. In this case, the holder of the put option makes a gain of $0.50 by exercising, and whoever sold them that put option makes a loss of $0.50 when the option is exercised against them.

35. Lenders to a firm can be modelled as owning a riskless bond and being short (i.e., having sold) a put option on the firm assets. The bond pays off a fixed $5 million in both scenarios.

36. In Scenario 1 (or any other scenario where the value of the firm is above the strike price of the face value of debt) the put option will expire without being exercised and the lenders just keep their $5 million bond payoff. The put option provides the holder with the right, but not the obligation, to sell the firm's assets for $5 million. If those assets are actually worth $9 million, the option to sell will not be exercised.

37. In Scenario 2, the put option will be exercised against the lenders. Exercise of the option means that the lenders are forced to pay the strike price of $5 million to buy the assets of the firm, which are worth only $3 million. This involves a loss of $2 million.

38. The put option component provides payoff of $0 in Scenario 1 and -$2 million in Scenario 2 – a standard short put payoff. The lenders have a combination of a riskless bond (which pays off $5 million in both Scenarios) and this short put option, so the net payoff to them is $5 million in Scenario 1 and $3 million in Scenario 2.

The contingent claims value is driven by the value of the firm

39. Just as with any derivative security, the value of the contingent claims on the firm are driven, among other things, by the value of the underlying asset. In the Merton model the underlying asset is the value of the firm assets, equity is modelled as a call option on the firm's assets and debt is modelled as the combination of a riskless bond and a short put over the firm's assets:

$$\begin{aligned} Value\ of\ firm &= Value\ of\ equity + Value\ of\ debt \\ &= Call\ option + (Riskless\ bond - Put\ option) \end{aligned}$$

40. The limited liability of equity means that no matter how bad the firm's performance, equity holders can walk away from firm's debt in exchange for payoff of zero. Limited liability is equivalent to equity holders issuing riskless debt but lenders giving equity holders a put on the firm's assets with a strike price equal to the face amount of the debt (the "default put").

41. The Merton framework then uses the Black-Scholes option pricing model to value the (default) put.

Factors that drive changes in the value of the assets of the firm will drive the returns to debt and equity in that firm

42. Since it can be clearly established that the value of both debt and equity are linked to the value of the assets of the firm, it must be the case that the factors that drive changes in the value of the assets of the firm also drive the returns to debt and equity in that firm. Thus, there must be an interrelationship between the return on debt and the return on equity in the same firm.

Foundational model

43. The Merton (1974) model forms the basis for all structural models and has been generalised and extended by the subsequent literature. The Merton model forms the basis of many models commonly used in practice, including Moody's KMV.[10] Here we discuss the basic model to illustrate the concepts in principal most clearly. It should be noted that subsequent developments in the literature have significantly enhanced this original framework, making the theoretical framework richer and better able to accommodate the key features of empirical data.

44. The Merton (1974) model follows the basic assumptions of the Black-Scholes model. That is, it assumes a lognormal distribution for the (maturity) value of the assets of the firm, deterministic interest rates, and constant volatility of firm value. The debt structure is assumed to consist entirely of a zero-coupon bond. The nature of bankruptcy is assumed to be costless and to preserve the strict priority of claims. Bankruptcy is assumed to be triggered at maturity should the value of the assets have fallen below the face value of debt. Some of the assumptions of the Merton model have been relaxed in subsequent work, as set out below.

Development of the Merton framework

45. The original Merton (1974) model is a parsimonious specification which is built on some key assumptions.[11] Consequently, it has been extended and refined, both in response to the results of

[10] Moody's KMV Model is the leading model of Kealhofer, McQuown and Vasicek of the KMV Corporation which was a financial technology firm pioneering the use of structural models for credit valuation. Moody's purchased the company in 2002.
[11] Note that most of the key assumptions of the model can be relaxed without altering the conclusions. The key assumptions relate to perfect capital markets, continuous time trading, the Modigliani-Miller theorem, a term structure that is flat and known with certainty, and that the dynamics of the value of the firm can be described by a diffusion-type stochastic process.

empirical testing and due to further analytical development. In this section, we discuss the problems have been faced and how these have been addressed and overcome by the subsequent literature.

No frictions assumptions

46. The Merton model assumes that in the event of default absolute priority holds, renegotiation is not permitted, and liquidation of the firm is costless.

47. Beginning within two years of the original paper, subsequent papers have extended the Merton framework considerably. Black and Cox (1976) develop an approach to valuing coupon-paying risky bonds with an endogenous default boundary, with and without asset sale restrictions, and demonstrate that safety covenants and asset sale restrictions can serve to improve the creditor's rights and increase debt values.

48. Leland (1994) advanced this framework by explicitly introducing taxes and bankruptcy costs[12]:

> This allowed, for the first time, a formal characterization of optimal capital structure, debt capacity, and credit spreads in a classic trade-off model. To do this, Leland explicitly introduced corporate taxes and bankruptcy costs, which may be interpreted as liquidation costs. Thus, he formalized the trade-off framework and provided a way to determine both (a) the optimal default boundary and (b) the value maximizing optimal capital structure.[13]

49. Subsequent papers provide even further sophistication. For example, Garlappi, Shu and Yan (2008) propose a model relating to shareholder bargaining power. Nielsen, Saá-Requejo, and Santa-Clara (2001), Longstaff and Schwartz (1995), and Mella-Barral (1999) are examples of the papers that relax the absolute priority assumption in their models.

50. Fundamentally though, these papers begin with the Merton framework and relax some assumptions to make the model more realistic. The key features of the strong link between the required returns on debt and equity are preserved as a central component of all of the papers that are discussed below.

Simplicity of the capital structure

51. The basic Merton model assumes a single zero-coupon bond debt structure.[14] Subsequent papers have developed adjustments that allow for the simultaneous existence of multiple debt issues that can differ in maturity, size of coupons and seniority. Black and Cox (1976) consider senior and subordinated debt, for example. Geske (1977) assumes that the firm's debt is a coupon bond instead of a zero-coupon one. In the model of Leland and Toft (1996) a firm will choose not only the amount but also the maturity of its debt.

52. The KV proprietary model of Moody's KMV is a generalization of the Merton model that allows for various classes and maturities of debt.[15]

53. Again, these papers begin with the Merton framework and relax some assumptions about the debt maturity structure. The key features of the strong link between the required returns on debt and equity are preserved after relaxing these assumptions.

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[12] Note that in doing so Leland relaxed the assumption of the Modigliani-Miller theorem that the value of the firm is invariant to its capital structure.

[13] Sundaresan (2013) at 5.6.

[14] Merton (1974) also considers the cases of coupon-bearing debt and callable debt and demonstrates that his model is capable of generating closed-form solutions for debt prices.

[15] See Bharath and Shumway (2008).

Default boundary and timing

54. The Merton model assumes that the default boundary is equal to the face value of the debt and that default can only occur at maturity. Again, the literature has expanded this initial approach. Black and Cox (1976) take an approach in which the firm's default occurs once the firm's asset value falls below a certain threshold. This can occur at any stage – it is not restricted to the maturity date of the debt. Consequently, the default time is now uncertain, ex ante. Further, equity is transformed from a European call option on the borrowing firm's assets into a down-and-out call option on the firm's assets.

55. Subsequent papers have explored various triggers for default. Some look at endogenous default triggers (where the equity holders choose to default or reorganize to maximize the value of their claims). These papers include, for example, Leland (1994); Leland and Toft (1996); and Acharya and Carpenter, (2002). Others employ exogenous default trigger models, which entail a pre-specified default barrier. These papers enhance the basic model by incorporating key characteristics of bond markets, such as stochastic risk-free interest rates (Longstaff and Schwartz, 1995), stochastic default barrier (Hsu, Saá-Requejo, and Santa-Clara, 2010), and mean-reverting leverage (Collin-Dufresne and Goldstein, 2001).

56. All of these papers begin with the Merton framework and relax some assumptions about the timing of potential defaults. The key features of the strong link between the required returns on debt and equity are preserved.

Default process

57. The diffusion type stochastic process assumed by Merton (1974) for the dynamics for the value of the firm, V, through time has been extended by the subsequent literature. More sophisticated structural models also incorporate jumps, (i.e., external shocks) (e.g., Zhou, 2001; Huang and Huang, 2012), stochastic volatility (e.g., Zhang, Zhou and Zhu, 2009; McQuade (2013) and regime-switching behaviour (e.g., Chun, Dionne and Francois, 2013, Hainaut and Colwell, 2014). The link between the required returns on debt and equity is preserved under the modified assumptions.

Interest rate process

58. Extensions have also been made to the original interest rate process assumed in Merton (1974). Stochastic interest rates have been modelled in numerous subsequent papers including, Shimko, Tejima and van Deventer (1993), Nielsen, Saá-Requejo, and Santa-Clara (1993), and Longstaff and Schwartz (1995).). The link between the required returns on debt and equity is preserved under the modified assumptions

Empirical examination of the Merton framework

59. The Merton framework has been used to examine some "puzzles" that have been identified in the empirical finance literature. In particular, insights from the Merton framework have provided explanations for these puzzles. Indeed, persistent empirical results that were considered to be puzzles in the context of other models have been shown to be perfectly consistent with (and even predicted by) a proper consideration of the Merton framework.

60. Once example is the "credit spread puzzle," which refers to the fact that actual market data show credit spreads that are much higher than those implied by structural models (by calibrating these models to historical default rates and recovery rates), across rating categories. For example, suppose the historical default rate for a particular type of rating is 1% and the recovery rate is 60%. This means that there is a 5% chance of a 40% loss. In this case, the yield would have to be 0.2% higher

to provide investors with the same expected return as a risk-free bond. The "puzzle" is that actual credit spreads tend to much higher than this.[16]

61. However, these "expected loss" characterisations fail to consider the fact that defaults are much more likely to occur in severe market downturns and therefore have a strong systematic element. The systematic nature of default risk means that investors do not simply require (risk-neutral) compensation for expected defaults, but they also require a risk premium. This very feature is illustrated in our simple two-state Merton framework in the subsequent section. We show, within the Merton framework, that a very low default probability can lead to a large default risk premium if there is systematic risk resulting from defaults being more likely to occur during market downturns.

62. In this regard, Feldhutter and Schaefer (2014) posit that the credit spread puzzle derives in large part from strong biases and low statistical power in commonly adopted approaches to testing the models. They suggest that the biases are due to Jensen's inequality and arise when tests are carried out on a representative firm rather than on individual firms. Using the Merton model and data on individual firms during 2002-2012 they quantify the size of the bias in spread predictions and find it to be particularly severe for high-quality firms and short-maturity bonds. Feldhutter and Schaefer (2014) test the Merton model via a bias-free approach using more than half a million transactions in the period 2002-2012 and find that the Merton model captures both the average level and time series variation of 10-year BBB-AAA spreads.

63. Another example of an empirical "puzzle" than can be explained by insights from the Merton framework is the so-called distress puzzle. Under the Merton (1974) model there is a positive relationship between credit risk and expected returns on equity. A number of empirical studies report that higher default probabilities are not associated with higher expected stock returns (where expected stock returns are typically assumed to be equal to realised returns). That is, some papers document a negative cross-sectional correlation between credit risk and future equity returns has been documented empirically (e.g., Dichev, 1998; Griffin and Lemon (2002); Campbell et al. 2008). These findings appear to suggest that investors pay a premium for bearing credit risk, which has led to the term "distress puzzle" being used to characterise these papers.

64. Consequently, much of the subsequent literature seeks to explain the mixed empirical findings. The key paper in that regard is Friewald, Wagner and Zechner (2013). One of their key findings is that the relationship between default risk requires consideration of both the physical *and* risk-neutral default probabilities. To see this, note that, other things being equal, a firm with higher expected returns on equity is less likely to default. Consequently, in the cross section we would expect to see a negative relationship between equity returns and physical default probabilities. This is not a "puzzle" at all. Also, the risk-neutral default probability is independent of the expected return on equity (it being the probability of default if the return on equity was equal to the risk-free rate). Consequently, other things being equal, one would expect to see no relationship between risk-neutral default probabilities and subsequent equity returns. It is only the combination of the two default probabilities that provides information about the market price of risk, and consequently about expected equity returns.

65. In this regard, Friewald, Wagner and Zechner (2013) state that:

> We first derive the link between equity risk premia and the dynamics of CDS spreads to show that a firm's expected excess returns and Sharpe ratios in the equity and credit markets are a function of the firm's physical *and* risk-neutral default expectations. Thus, exclusively relying on the firm's CDS spreads directly observable in the market *or* on

[16] An early paper to document this was Jones, Mason & Rosenfeld (1984).

> estimates of the firm's physical default probability is not sufficient to understand the relation between a firm's credit risk and its equity returns. [17]

and they go on to conclude that:

> Our analysis also generates insights for the distress puzzle – the finding that firms with high distress risk earn anomalously low equity risk premia (see, for example, Campbell, Hilscher, and Szilagyi (2008)). We show that the distress puzzle is perfectly consistent with structural models and find that empirically, firms with the highest physical or risk-neutral default probabilities earn the lowest equity returns. These findings complement our results documenting that there is a strong positive relation between credit risk premia and subsequent equity returns. [18]

66. In summary, Friewald, Wagner and Zechner (2013) explain that the distress puzzle is actually no puzzle at all. Rather, it is entirely consistent with the standard Merton framework – in which required returns on debt and equity are positively correlated.

67. In summary, a number of empirical "puzzles" such as the credit spread puzzle and the distress puzzle can be explained by the insights that are available from the Merton framework. Indeed, these empirical results were only ever deemed to be puzzles because some authors had failed to fully appreciate the rich insights that could be drawn from the Merton framework.

[17] Friewald, Wagner and Zechner (2013), p. 5.
[18] Friewald, Wagner and Zechner (2013), p. 26.

4. Modern application of the contingent claims framework

Overview

68. The modern application of the Merton (1974) contingent claims framework focuses on the relationship between the required return on equity and the required return on debt. In particular, one of the key insights of the Merton framework is that the equity risk premium and the debt risk premium must be linked by the following equation:

$$E[r_e] - r_f = \Omega_{e,d}\left(E[r_d] - r_f\right) \tag{1}$$

where the key term is the elasticity of equity relative to debt:

$$\Omega_{e,d} = \frac{\partial E/E}{\partial D/D}.$$

69. For example, Campello, Chen and Zhang (2008), which is published in the top-ranking *Review of Financial Studies*, explain that:

> Because both equity and debt are contingent claims written on the same productive assets, a firm's equity risk premium is naturally tied to its debt risk premium. [The equation above] formalizes this argument: the equity risk premium equals the debt risk premium multiplied by the elasticity of the equity value with respect to the bond value…the equity value and the bond value are driven only by the asset value. Our framework still allows multiple common factors, but they affect equity and bond values through the firm value. [19]

70. That is, the key insight from the Merton framework is that equity and debt are contingent claims over the assets of the same firm. These two investor classes will share the payoffs from the same firm between them. Consequently there must be some relationship between the returns to each. As Campello, Chen and Zhang (2008) explain above, the Merton framework is agnostic about how assets are valued. Merton takes no view about whether assets are valued in accordance with the CAPM or a multi-factor model. Rather, the Merton framework takes the value of the firm's assets as given, and focuses on the relativity of the return that those assets are expected to provide to equity holders and the return that those same assets are expected to provide to debt holders.

71. Friewald, Wagner and Zechner (2013), which has been accepted for publication in the *Journal of Finance*, also examine the relationship between returns on debt and equity within the Merton framework. They show that within the Merton framework, the elasticity is equal to the ratio of the volatilities:

$$\Omega_{e,d} = \frac{\sigma_e}{\sigma_d}.$$

72. Appendix 1 to this report explains the derivation of the results set out above within the context of a simple two-state economy. That appendix develops some intuition for the Merton framework in a simple economy and shows that the results above are consistent with each other and are all perfectly consistent with the Sharpe-Lintner CAPM. In particular, the appendix shows that Equation (1) holds in the case where the expected return on equity and the expected return on debt are both entirely consistent with the Sharpe-Lintner CAPM.

[19] Campello, Chen and Zhang (2008), p. 1302.

Relevance to the regulatory framework

73. As set out in Section 2 above, the linkage between the required returns on debt and equity in the same benchmark firm appears to be central to the NGR 87(5) requirements to have regard to all relevant evidence, consistency, and interrelationships between parameters for equity and debt. The Merton model provides the standard framework for modelling the linkage between the required returns on debt and equity in the same firm. The Merton framework shows that there are clear linkages between the required return on equity, the required return on debt, the elasticity between equity and debt and the relative volatilities of equity and debt.

74. This framework can be used in the regulatory setting as a check of the consistency between the allowed return on equity, a check of the interrelationships between parameters that are common to the return on equity and the return on debt, and as a check on the overall reasonableness of the allowed return on equity relative to the allowed return on debt. These checks can be performed in a number of ways. For example:

    a) An allowed return on debt and an empirical estimate of elasticity jointly provide information about what would be a reasonable range for the required return on equity; and

    b) An allowed return on debt and an allowed return on equity jointly imply a particular elasticity, which can then be tested against elasticity benchmarks for the regulated firm.

75. Before proceeding, we reiterate the very important point that we have made above. We do not suggest that this framework can be used to obtain a single point estimate of the required return on equity from the analysis of primary data. Estimating the required return on equity is a complex task that requires consideration of a whole range of models, estimation methods, data and other evidence. Rather, our point is that the Merton framework is very useful when considering the relationship between the required return on equity and the required return on debt for the same firm. The Merton framework provides valuable insights into the *relativity* between these two quantities. The Merton framework has been shown to perform well empirically in explaining the relative returns on equity on debt and it is for that purpose that we consider in this report. The relativity between the required return on equity and the required return on debt takes on new importance under NGR 87(5) which requires stakeholders to have regard to the consistency of parameter estimates and to the interrelationships between parameters.

Estimation

76. In a recent paper in the top-ranking *Journal of Financial Economics*, Schaefer and Strebulaev (2008) empirically examine the ability of the Merton model to explain the relationship between equity and debt risk premiums in the same firm. In the course of this work, they provide an accessible derivation of what they call the "hedge ratio" which is simply the inverse of the Merton elasticity:

$$\mathrm{h}_e = \frac{\partial D/D}{\partial E/E} = \frac{1}{\Omega_{e,d}}.$$

77. They show that, within the context of the Merton framework:

$$\mathrm{h}_e = \left(\frac{1}{\Delta} - 1\right)\left(\frac{1}{L} - 1\right) \tag{2}$$

where Δ is the derivative of equity value with respect to the value of the assets of the firm and L is the market value leverage.

78. Within the Merton framework, the Δ parameter is in the form of the standard call option delta originally derived by Black and Scholes (1973):

$$\Delta = \mathrm{N}(d_1)$$

where $\mathrm{N}(\bullet)$ represents the cumulative normal distribution function and:

$$d_1 = \frac{ln(V/D) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}$$

where V is the value of the firm, D is the face value of the firm's debt, T is the time to maturity of the firm's debt, and σ is the volatility of the return on the firm's assets.

79. Thus, the hedge ratio in Equation (2) above varies indirectly with the equity delta and with the market value leverage. Consequently the Merton elasticity (which is the inverse of the hedge ratio) varies directly with the equity delta and market value leverage. The intuition for these relationships is as follows:

    a) The equity delta measures the sensitivity of the value of equity to the value of the firm's assets for a given level of debt finance. Thus, a higher delta means that the firm's equity is more sensitive to changes in the value of the firm's assets, other things being equal; and

    b) As market value leverage increases, equity becomes more of a residual claim with more debt ranking ahead of it on a market value basis. This levers up the equity claim making it relatively more sensitive to changes in the value of the firm's assets.

80. Schaefer and Strebulaev (2008) conclude that this Merton hedge ratio fits the empirical data extremely well. They conclude that the Merton model:

> Provide[s] quite accurate predictions of the sensitivity of corporate bond returns to changes in the value of equity (hedge ratios)[20]

and that:

> Our main results support the view that structural models account well for the credit-related component of corporate bond prices, and we find that even the simple Merton (1974) model produces hedge ratios to equity that are remarkably consistent with those observed empirically. [21]

81. We use the Schaefer and Strebulaev (2008) approach to estimate Merton hedge ratios for a range of input parameters. We then take the inverse of the hedge ratio to produce an estimate of the Merton elasticity, as used by Campello et al as set out above. Our elasticity estimates are summarised in Figure 1 to Figure 3 below. Our base case parameter estimates are as follows:

    a) Risk-free rate of 3.6% (the current 10-year government bond yield);

    b) Amount of debt equal to 60% of the value of the assets of the firm, as per the regulatory benchmark;

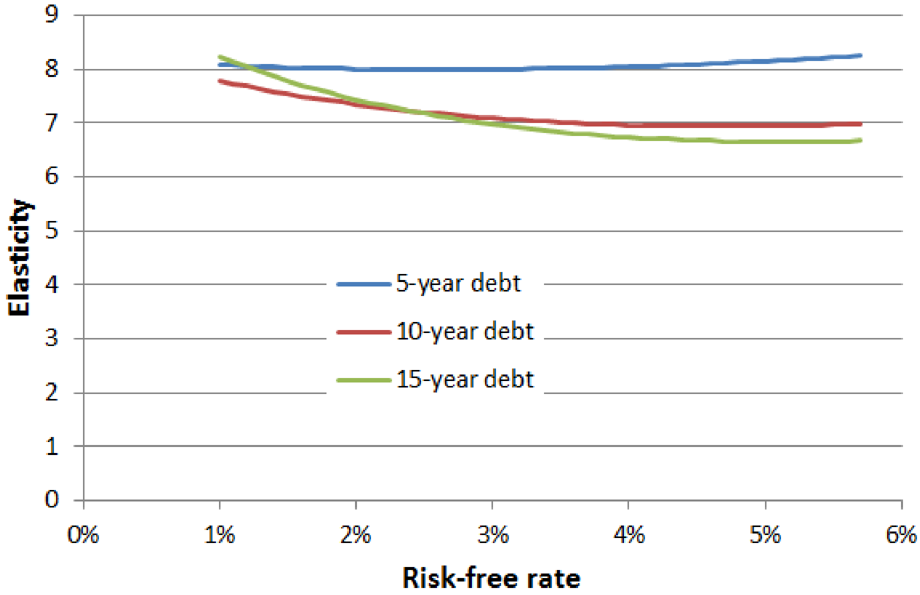
[20] Schaefer and Strebulaev (2008), p. 1.
[21] Schaefer and Strebulaev (2008), pp. 2-3.

c) Term of debt of 10 years, as per the regulatory benchmark;

d) Volatility of 40%, as per the historical volatility of domestic benchmark comparator firms.

82. Figure 1 shows that the elasticity is sensitive to the term of debt and to the risk-free rate. In particular, for long-term debt the elasticity rises as the risk-free rate falls. The dominant effect here is that a fall in the risk-free rate increases the market value of the firm's debt (for a given face value of debt, and all other things being equal). As explained in Paragraph 79 above, elasticity rises with increases in the market value leverage of the firm. For shorter-term debt, changes in the risk-free rate have a relatively smaller effect on the market value of the firm's debt because the compounding effect of a change in interest rates applies over a shorter period. For short-term debt, the dominant effect comes via the equity delta, which increases marginally with increases in the risk-free rate. The reason for this effect is that call options increase in value as increases in the risk-free rate decrease the present value of the strike price.

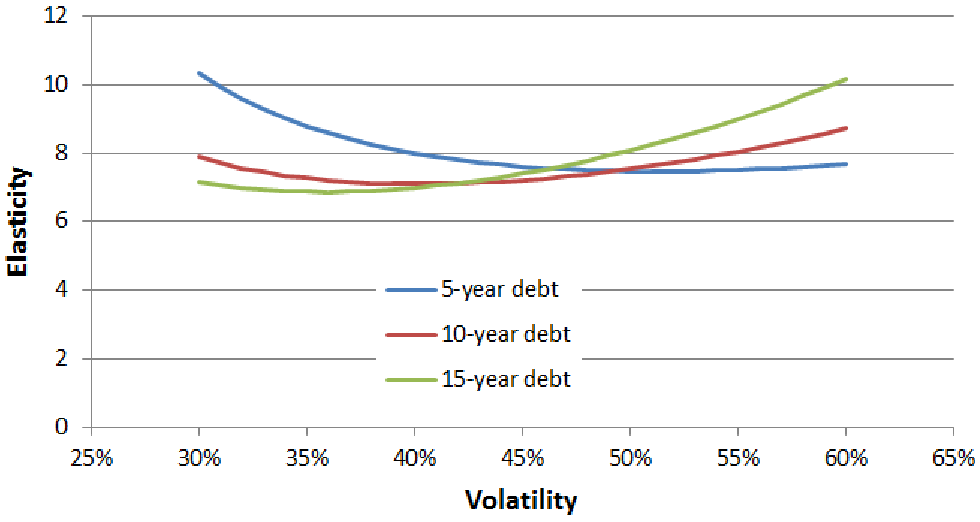
Figure 1
Merton elasticity by term of debt and risk-free rate



SFG calculations from Merton (1974). Volatility set to 40%, debt set to 60% of firm value.

83. Figure 2 shows that the elasticity is also sensitive to the volatility of the returns on the firm's assets. For longer-term debt, the elasticity is generally higher for more volatile firms. The dominant effect here is that for long-term debt higher volatility equates to a higher probability of the equity call option finishing in the money, hence higher volatility is associated with higher Δ and therefore higher elasticity. For short-term debt the reverse is true – since the equity call option is in the money, high volatility over a short term serves to decrease the probability of the equity option finishing in the money (especially for low risk-free rates).

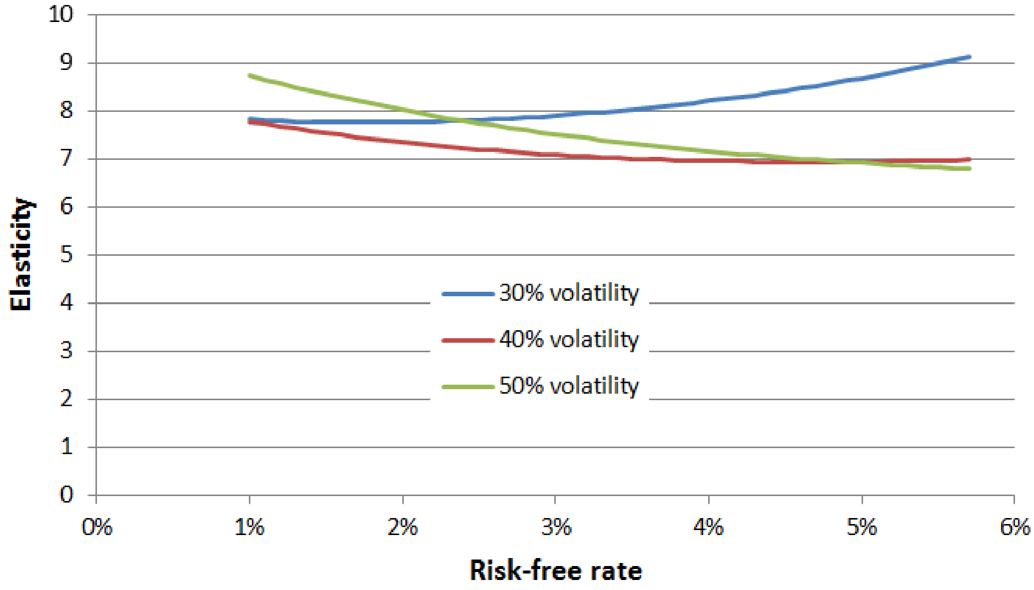
Figure 2
Merton elasticity by term of debt and volatility



SFG calculations from Merton (1974). Risk-free rate set to 3.6%, debt set to 60% of firm value.

84. Figure 3 shows that, for debt of 10 years, the elasticity is sensitive to the risk-free rate and volatility. As above, the elasticity generally rises as the risk-free rate falls. The dominant effect here is again that a fall in the risk-free rate increases the market value of the firm's debt (for a given face value of debt, and all other things being equal). For low volatility assets, the elasticity falls as the risk-free rate falls. This is because the probability of the equity finishing in the money (Δ) falls as the risk-free rate (drift) falls for low volatility assets.

Figure 3
Merton elasticity by volatility and risk-free rate



SFG calculations from Merton (1974). Term of debt set to 10 years, debt set to 60% of firm value.

85. In all of the cases above, we maintained the level of debt at 60% of the value of the firm's assets. The figures above show that there is no reasonable combination of parameters that produces an

elasticity parameter value below 6.0. This places a constraint on the relativity between the expected returns on debt and equity, as set out in the following section.

Application to the regulatory framework

86. We begin with the key insight of the Merton framework set out above – that the equity risk premium and the debt risk premium (for the same firm) must be linked by the following equation:

$$E[r_e] - r_f = \Omega_{e,d}\left(E[r_d] - r_f\right)$$

where the key term is the elasticity of equity relative to debt:

$$\Omega_{e,d} = \frac{\partial E/E}{\partial D/D}.$$

87. Our intention is to use the Merton framework as a reasonableness cross check of the equity risk premium for the benchmark efficient firm, $E[r_e] - r_f$, given the expected debt risk premium efficient firm, $E[r_d] - r_f$. We do this by selecting values at the lower bound of the reasonable range for $\Omega_{e,d}$ and $E[r_d]$. These lower bound values then produce a lower bound for the equity risk premium. The equity risk premium must be greater than this lower bound in order to be consistent with the debt risk premium. We interpret this exercise as a cross check on the consistency between a pair of equity and debt risk premiums. If the equity risk premium is below the lower bound estimate, we conclude that it is not internally consistent with the debt risk premium.

88. Our first task then is to compute reasonable lower bound estimates for the elasticity and the expected debt risk premium. As set out in Figure 1 to Figure 3 above, there is no combination of parameters that produces an elasticity estimate below 6. Consequently we adopt 6 as a reasonable lower bound for elasticity.

89. Next we note that our cross check requires an estimate of the expected debt risk premium, $E[r_d] - r_f$. The ERA sets the 10-year cost of debt to 5.2%, which includes a spread of 1.8%.[22] This spread is for a generic BBB+ 10-year bond, for which there is a small but positive probability of default each year. Consequently, the 1.8% represents a promised spread and not an expected spread. The promised spread can be converted into the expected spread via a deduction for expected default.

90. We obtain a lower bound for the expected debt risk premium by selecting an upper limit for the deduction for expected default. In particular, if the deduction for expected default is large, $E[r_d]$ becomes smaller, the expected debt risk premium is lower, and a lower equity risk premium is required.

91. To select an upper limit for the deduction for expected default, we note that Standard & Poor's (2013) report that the default rate on BBB+ corporate bonds has been 0.15% per year over the last 30 years. A ten-year bond has ten (annual) chances to default with a 0.15% probability assigned to each chance. Thus, the probability of not defaulting at all during the 10-year term is $(1-0.15\%)^{10} - 1 = 98.5\%$. Thus, the probability of a default occurring at some point during the ten year life of the bond is 1.5%.[23]

[22] ERA ATCO Gas Draft Decision, p. 210, Paragraph 892.

[23] The fact that the 10-year default probability is ten times the one-year default probability in this case is a coincidence. This calculation could be performed in a more complex way by allowing for ratings transitions during the ten years, however that approach adds considerable complexity and is unlikely to produce materially different outcomes.

92. Next we note that Standard and Poor's (2013) report an average recovery rate of 50% for BBB+ corporate bonds. That is, on average, when BBB+ bonds default 50% of the face value of the bond is returned to investors, who lose the other 50%.

93. Drawing together the probability of default and the loss given default allows us to compute the expected return on debt:

$$E(r_d) = 0.985 \times 5.2\% + 0.015 \times -50\% = 4.4\% .$$

94. That is, the adjustment for expected default is 0.8% (5.2%-4.4%). Consequently, the adjusted debt spread, or the expected return on debt, is 1% (1.8%-0.8%).

95. In our view, this is a lower bound for the expected return on debt for the benchmark efficient firm. In particular, it would not seem to be open to a regulator to argue that they have set the allowed prices at such a low level that there is a material chance of the regulated firm becoming bankrupt.

96. Drawing all of the above information together yields our lower bound reasonableness test:

$$E[r_e] - r_f > \Omega_{min}\left(E[r_d] - r_f\right)_{min}$$

$$E[r_e] - r_f > 6.0 \times 1.0\% = 6.0\%.$$

97. That is, given the debt risk premium, internal consistency requires that the equity risk premium must be at least 6.0%.

Application to the ERA's Guideline

98. The equity risk premium proposed in the ERA's Guideline is only 4.2%.[24] To reduce the lower bound for the equity risk premium to the ERA's 4.2% would require:

    a) Reducing the elasticity to 4.2; or

    b) Reducing the expected debt risk premium to 70 basis points.

99. Neither of these are reasonable assumptions. Setting the elasticity to 4.2 is simply impossible – there is no combination of parameters that makes this possible. Setting the expected debt risk premium to 70 basis points suggests that debt investors expect to receive a return of only 0.7% above the base risk-free rate. This is only possible if the expected probability of default is 40% higher than the observed empirical default probability, which would be an unreasonable assumption in our view. Indeed there has been no rolling 10-year period any time in the last 20 years with a default rate as high as that which would be required to support an expected debt risk premium as low as 0.7% – even including the peak of the GFC over 2008 and 2009.

100. Our conclusion is that the Merton framework provides a test of the internal consistency of the allowed returns on debt and equity and that the allowed returns on debt and equity in the ERA's Guideline fail that test – the allowed return on equity is too low to be consistent with the allowed return on debt.

[24] 0.7×6.0%.

Application to the ATCO Gas Draft Decision

101. The equity risk premium proposed in the ATCO Gas Draft Decision is only 3.8%[25] To reduce the lower bound for the equity risk premium to the ERA's 3.8% would require:

   a) Reducing the elasticity to 3.8; or

   b) Reducing the expected debt risk premium to 60 basis points.

102. Again, neither of these are reasonable assumptions. Setting the elasticity to 3.8 is simply impossible – there is no combination of parameters that makes this possible. Setting the expected debt risk premium to 60 basis points suggests that debt investors expect to receive a return of only 0.6% above the base risk-free rate. This is only possible if the expected probability of default is 50% higher than the observed empirical default probability, which would be an unreasonable assumption in our view.

103. The ATCO Gas Draft Decision has reduced the allowed equity risk premium even further relative to the ERA's Guideline. This makes it even more inconsistent with the given level for the allowed return on debt, within the Merton framework.

Application to the DBP submission

104. The same cross check for internal consistency between the equity and debt risk premiums can be applied to the DBP submission. We are instructed that DBP propose a total cost of debt within the range of 5.66% to 5.77% (net of any new issue premium and the 15 basis points for debt issuance costs) and we apply the following calculations:

   a) We subtract the DBP estimate of the risk-free rate of 3.54% to produce a range for the debt risk premium of 2.13% to 2.24%;

   b) We then subtract the same 0.82% adjustment for expected default as above[26] to produce a range for the expected debt risk premium, $E[r_d] - r_f$, of 1.31% to 1.42%;

   c) We then multiply the expected debt risk premium by our lower bound elasticity estimate of 6 to produce a range for the equity risk premium of 7.86% to 8.52%.

105. In our view, an appropriate lower bound for the equity risk premium, $E[r_e] - r_f$, is within the range of 7.86% to 8.52%. Within the context of the Merton framework, an equity risk premium that is within this range would be consistent with the debt risk premium range that has been proposed by DBP. We note that this is a conservative range for the equity risk premium, given that it is based on a lower bound estimate of elasticity.

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[25] 0.7×5.5%.

[26] We continue to note that we consider this to be a conservative upper bound for the adjustment for default that is consistent with a lower bound for the estimate of the equity risk premium.

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Appendix 1: A simple illustration of the Merton framework

Overview

106. This appendix sets out a simple illustration of the key results from the Merton framework within the context of a simple two state economy. This setting involves a one-period economy with two states. In the "up" state, the broad market and the firm in question both increase in value. In the "down" state, the market falls, the assets of the firm decline in value and the firm defaults on its debt obligations. The Merton (1974) model is derived in continuous time with an infinite number of states. The two-state illustration in this appendix is designed to illustrate the key features and key results of the Merton framework in a simple and easy to understand setting.

The two-state economy

107. In our simple one-period, two-state model, the risk-free rate is 5% and the broad market and the firm in question have uncertain prospects as set out in Figure 4 below.

Market portfolio

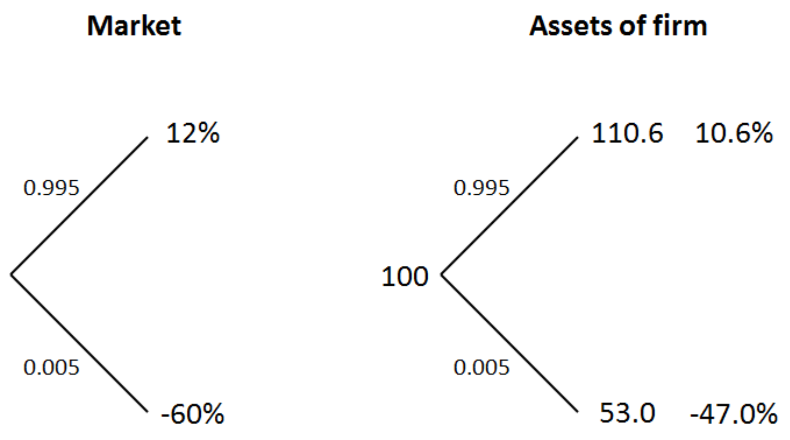
108. In this example, the broad market has a 99.5% probability of delivering a 12% return and there is a 0.5% (one in 200 years) probability of a severe crash in which 60% of the value of the market is lost (similar to what happened to the Australian market over the 1970-71 period). Relevant statistics for the market portfolio are as follows:

    a) The expected return of the market is 11.64%;[27]

    b) The market risk premium is 6.64%;[28] and

    c) The standard deviation of market returns is 5.08%.[29]

Figure 4
Example market and firm assets



[27] $0.995 \times 12\% + 0.005 \times -60\% = 11.64\%$.

[28] $11.64\% - 5\% = 6.64\%$.

[29] $\sqrt{0.995 \times (12\% - 11.64\%)^2 + 0.005 \times (-60\% - 11.64\%)^2} = 5.08\%$.

Assets of the firm

109. The assets of the firm have an initial value of \$100 and will increase to \$110.60 (a 10.6% return) if the market rises, and will fall to \$53 (a -47% return) if the market falls. Relevant statistics for the value of the firm's assets are as follows:

a) The expected return of the firm's assets is 10.31%;[30]

b) The standard deviation of asset returns is 4.06%;[31]

c) The covariance between asset returns and market returns is 0.21%;[32] and

d) The asset beta is 0.8.[33]

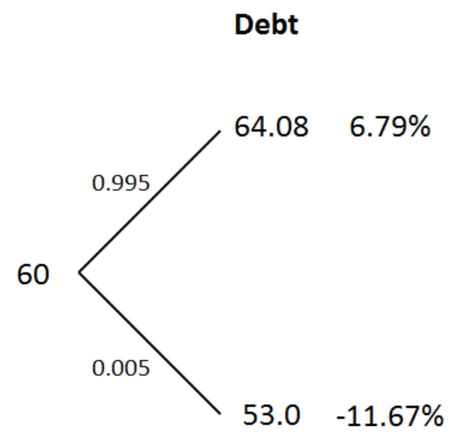
110. Note that the expected return is consistent with the Sharpe-Lintner CAPM in this setting:

$$\begin{aligned} E[r_a] &= r_f + \beta_a \times MRP \\ &= 5\% + 0.8 \times 6.64\% = 10.31\%. \end{aligned}$$

Debt

111. In this example, 60% of the firm's assets are financed with debt. The promised yield on this debt is 6.79%. Consequently, if the firm's assets turn out to be worth \$110.60 (99.5% probability), the debt holders will be paid the \$64.08 that they have been promised and their return will be 6.79%. In the event that the assets of the firm turn out to be worth only \$53 (0.5% probability), the firm will default and the debt holders will receive the assets of the firm and their return will be -11.67%. These figures are summarised in Figure 5 below.

Figure 5
Example debt values and returns



112. Relevant statistics for the firm's debt are as follows:

a) The expected return on the firm's debt is 6.70%;[34]

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[30] $0.995 \times 12\% + 0.005 \times -60\% = 11.64\%.$

[31] $\sqrt{0.995 \times (10.6\% - 10.31\%)^2 + 0.005 \times (-47\% - 10.31\%)^2} = 4.06\%.$

[32] $0.995 \times (10.6\% - 10.31\%)(12\% - 11.64\%) + 0.005 \times (-47\% - 10.31\%)(-60\% - 11.64\%) = 0.21\%.$

[33] $\beta_a = \frac{Cov(r_a, r_m)}{Var(r_m)} = \frac{0.21\%}{5.08\%^2} = 0.8.$

b) The standard deviation of returns on debt is 1.30%;[35]

c) The covariance between debt returns and market returns is 0.07%;[36] and

d) The debt beta is 0.26.[37]

113. Note that the expected return on debt is also consistent with the Sharpe-Lintner CAPM in this setting:

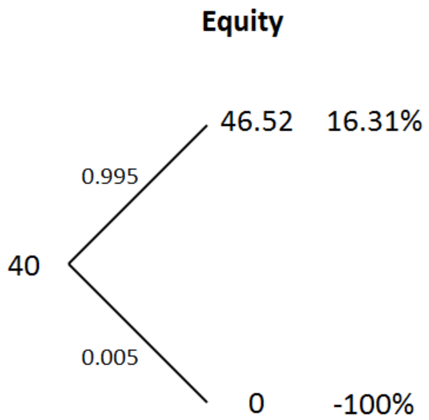
$$\begin{aligned} E[r_d] &= r_f + \beta_d \times MRP \\ &= 5\% + 0.26 \times 6.64\% = 6.70\%. \end{aligned}$$

114. In this example, the debt risk premium (i.e., the difference between the promised yield and the risk-free rate) is 179 basis points.[38] The DRP is substantial in this example, even though the probability of default is remote (one in 200 years). The reason for this is the systematic nature of default – defaults tend to occur during market downturns, not expansions.[39] In this example, the expected return and the promised yield differ by only 9 basis points. That is, the promised yield must be reduced by only 9 basis points to obtain the expected return on debt.

Equity

115. In this example, 40% of the firm's assets are financed with equity. The equity holders receive a residual cash flow – whatever the assets of the firm ore worth, less the payment made to debt holders. Consequently, if the firm's assets turn out to be worth $110.60 (99.5% probability), the debt holders will be paid the $64.08 that they have been promised and the equity holders will receive the remaining 46.52, a return of 16.31%. In the event that the assets of the firm turn out to be worth only $53 (0.5% probability), the firm will default and the equity holders will receive nothing. These figures are summarised in Figure 6 below.

Figure 6
Example equity values and returns



[34] $0.995 \times 6.79\% + 0.005 \times -11.67\% = 6.70\%$.

[35] $\sqrt{0.995 \times (6.79\% - 6.70\%)^2 + 0.005 \times (-11.67\% - 6.70\%)^2} = 1.30\%$.

[36] $0.995 \times (6.79\% - 6.70\%)(12\% - 11.64\%) + 0.005 \times (-11.67\% - 6.70\%)(-60\% - 11.64\%) = 0.07\%$.

[37] $\beta_d = \frac{Cov(r_d, r_m)}{Var(r_m)} = \frac{0.07\%}{5.08\%^2} = 0.26$.

[38] $6.79\% - 5\% = 1.79\%$.

[39] The systematic nature of defaults is incorporated into numerous papers including Anginer & Yildzhan (2013), Avramov et al. (2009), and Giesecke et al. (2011).

116. Relevant statistics for the firm's equity are as follows:

a) The expected return on the firm's equity is 15.73%;[40]

b) The standard deviation of returns on equity is 1.30%;[41]

c) The covariance between equity returns and market returns is 0.42%;[42] and

d) The equity beta is 1.62.[43]

117. Note that the expected return on equity is also consistent with the Sharpe-Lintner CAPM in this setting:

$$\begin{aligned} E[r_e] &= r_f + \beta_e \times MRP \\ &= 5\% + 1.62 \times 6.64\% = 15.73\%. \end{aligned}$$

118. It is important to note in this case that, in the regulatory setting, the *allowed* return on equity would have to be set at 16.31% – which is higher than the *expected* return from the Sharpe-Lintner CAPM of 15.73%. If the regulator sets the allowed return on equity at 16.31%, the equity holders will earn that return only so long as the firm does not default. There is a small probability that the firm will default and the equity will be worthless. The probability weighted average of these two possibilities provides an expected return of 15.73%. If the regulator allowed a return of 15.73%, the *expected* return would be:

$$E[r_e] = 0.995 \times 15.73\% + 0.005 \times -100\% = 15.15\%,$$

which is less than the expected return from the CAPM.

Elasticity

119. The elasticity of equity with respect to debt in this example is given by:

$$\Omega_{e,d} = \frac{\Delta E/E}{\Delta D/D} = \frac{(46.52 - 0)/40}{(64.08 - 53.00)/60} = 6.3.$$

The relationship between debt and equity returns

120. As set out in Equation (1) above, Campello, Chen and Zhang (2008) exploit a key insight from the Merton (1974) framework that links equity and debt risk premiums:

$$E[r_e] - r_f = \Omega_{e,d}\left(E[r_d] - r_f\right)$$

121. This implies that the expected return on equity can be estimated as:

$$\begin{aligned} E[r_e] &= r_f + \Omega_{e,d}\left(E[r_d] - r_f\right) \\ &= 5\% + 6.30(6.70\% - 5\%) = 15.73\%. \end{aligned}$$

[40] $0.995 \times 16.31\% + 0.005 \times -100\% = 15.73\%$.

[41] $\sqrt{0.995 \times (16.31\% - 15.73\%)^2 + 0.005 \times (-100\% - 15.73\%)^2} = 8.20\%$.

[42] $0.995 \times (16.31\% - 15.73\%)(12\% - 11.64\%) + 0.005 \times (-100\% - 15.73\%)(-60\% - 11.64\%) = 0.42\%$.

[43] $\beta_e = \frac{Cov(r_e, r_m)}{Var(r_m)} = \frac{0.42\%}{5.08\%^2} = 1.62$.

122. That is, the expected return on equity can be estimated as a function of the expected return on debt and the elasticity of equity with respect to debt.

The market price of risk and volatility

123. Friewald, Wagner and Zechner (2013), which has been accepted for publication in the *Journal of Finance*, also examine the relationship between returns on debt and equity within the Merton framework. They begin by defining the *market price of risk* for the assets of the firm, or the firm's *Sharpe ratio* in the standard manner as: [44]

$$\lambda_a = \frac{E[r_a] - r_f}{\sigma_a}.$$

124. In the numerical example above, we have:

$$\lambda_a = \frac{10.31\% - 5\%}{1.30\%} = 1.31.$$

125. One of the key insights of Merton (1974) is that equity in a levered firm is effectively a call option over the firm's assets with strike price equal to the face value of the debt. This allows some well-known option pricing results to be applied to equity. In particular, the expected excess return on a call option is equal to the expected excess return on the underlying asset multiplied by the elasticity of the option with respect to the underlying asset. That is: [45]

$$E[r_e] - r_f = \left(E[r_a] - r_f\right)\Omega_{e,a}. \qquad (3)$$

where

$$\Omega_{e,a} = \frac{\partial E/E}{\partial A/A}.$$

126. In the example above, we have:

$$\Omega_{e,a} = \frac{(46.52 - 0)/40}{(110.60 - 53)/100} = 2.02,$$

and:

$$E[r_e] = r_f + \left(E[r_a] - r_f\right)\Omega_{e,a}$$
$$15.73\% = 5\% + (10.31\% - 5\%)2.02.$$

127. Another standard option pricing result is that the volatility of equity must be equal to the volatility of the underlying asset multiplied by the elasticity of equity relative to the assets of the firm: [46]

$$\sigma_e = \sigma_a \Omega_{e,a}. \qquad (4)$$

[44] Friewald, Wagner and Zechner (2013), Equation 1.
[45] Friewald, Wagner and Zechner (2013), Equation 2.
[46] Friewald, Wagner and Zechner (2013), Equation 3.

128. In the example above, we have:

$$8.20\% = 4.06\% \times 2.02.$$

129. Equations (3) and (4) imply that the market price of risk for equity must equal the market price of risk for the underlying asset:

$$\lambda_e = \frac{E[r_e] - r_f}{\sigma_e} = \frac{\left(E[r_a] - r_f\right)\Omega_{e,a}}{\sigma_a \Omega_{e,a}} = \lambda_a.$$

130. In the example above, we have:

$$\lambda_e = \frac{E[r_e] - r_f}{\sigma_e} = \frac{15.73\% - 5\%}{8.2\%} = 1.31 = \lambda_a.$$

131. The same analysis can be applied to debt securities to show that those securities must also have the same market price of risk.[47] In the example above, we have:

$$\lambda_d = \frac{E[r_d] - r_f}{\sigma_d} = \frac{6.70\% - 5\%}{1.3\%} = 1.31 = \lambda_a = \lambda_e. \qquad (5)$$

132. At this point, Friewald, Wagner and Zechner (2013) define the generic market price of risk (which is the same for the assets of the firm and equity and debt securities) to be λ. Consequently:

$$\frac{E[r_e] - r_f}{\sigma_e} = \lambda,$$

in which case:

$$E[r_e] = r_f + \lambda\sigma_e, \qquad (6)$$

which is the key result that forms the basis of the empirical tests of Friewald, Wagner and Zechner (2013).

133. We note that this result is entirely consistent with the key relationship derived by Campello, Chen and Zhang (2008), which is not surprising given that both are based on the standard Merton framework. Starting with Equation (4) above, and substituting in the definitions of λ_d from Equation (5) and noting that:

$$\sigma_e = \sigma_a \frac{\partial E/E}{\partial A/A} \text{ and } \sigma_d = \sigma_a \frac{\partial D/D}{\partial A/A}$$

yields:

$$E[r_e] = r_f + \lambda\sigma_e$$

[47] Friewald, Wagner and Zechner (2013),Equations 5 and 6. Note that we have considered standard debt securities in our analysis above. Friewald, Wagner and Zechner (2013) consider credit default swaps, which have a payoff that is the reverse of standard debt securities (i.e., they pay nothing if the firm remains solvent, but pay out the amount that would be lost by debt holders in the event that the firm defaults).

$$= r_f + \frac{E[r_d] - r_f}{\sigma_a \frac{\partial D/D}{\partial A/A}} \times \sigma_a \frac{\partial E/E}{\partial A/A}$$
$$= r_f + \left(E[r_d] - r_f\right) \times \frac{\partial E/E}{\partial D/D}$$
$$= r_f + \left(E[r_d] - r_f\right) \times \Omega_{e,d}$$

which is the key result from Campello, Chen and Zhang (2008) in Equation (1) above with:

$$\Omega_{e,d} = \frac{\sigma_e}{\sigma_d}.$$

Appendix 2: Instructions

Ensuring consistency between debt and equity

The Rules require that the return on debt and equity be consistent, as per Rule 87(5):

> *"In determining the allowed rate of return, regard must be had to:*
> *(a) relevant estimation methods, financial models, market data and other evidence;*
> *(b) the desirability of using an approach that leads to the consistent application of any estimates of financial parameters that are relevant to the estimates of, and that are common to, the return on equity and the return on debt; and*
> *(c) any interrelationships between estimates of financial parameters that are relevant to the estimates of the return on equity and the return on debt."*

Regulators have noted the issue of consistency between the premia for debt and equity in excess of the risk-free rate, and the ERA has noted in the appendices to its Rate of Return Guidelines (Appendix 29) that the relative differences between the spreads for debt and equity may be relevant information, but has not sought to formalise this notion.

DBP is aware of a literature in the options pricing literature stemming from a seminal paper by Merton (1974), which notes that debt and equity are options on the same underlying asset, and proceeds thence to developing an understanding of the mathematical relationship which ought to exist between the two.[48]

The required task

DBP is looking for expert opinion which consists of two distinct parts. The first part is a requirement submitting a response to this request for tender. The second part is a detailed literature review, to understand how this part of the options-pricing literature has developed, and how it might be relevant to the question of consistency as it exists under the Rules.

The second stage of the project operationalizes the model which has been developed, based upon the literature review. This may be some variant of the models footnoted above, or some other model.

[48] See Merton, R, 1974, "On the Pricing of Corporate Debt: The risk structure of interest rates", Journal of Finance, 29, pp.449-47, and Galai, D & Masulis, RW, 1976, "The Option pricing Model and the Risk Factor of Stock", Journal of Financial Economics, 3, 53-81, as two early papers in this literature, and SchaeferSM & Strebulaev, IA, 2008, "Structural Models of Credit Risk are Useful: Evidence from hedge ratios on corporate bonds", Journal of Financial Economics, 90, 1-19 andCampello, M, Chen, L & Zhang, L, 2008, "Expected Returns, Yield Spreads and Asset Pricing Tests", Review of Financial Studies, 21(3), 1297-1339 as two more recent contributions.