REVIEW OF SFG'S APPROACH TO ESTIMATING THE COST OF EQUITY

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EXECUTIVE SUMMARY

This paper has addressed a number of issues raised with me by the ERAWA, and my views are as follows.

Firstly, SFG's theoretical analysis significantly diverges from standard finance theory in using state prices with the market portfolio as the 'underlying' asset to the value of a firm, and also in assuming payoffs from regulated assets only at five-yearly frequencies. In addition, their analysis is wrong in applying a formula to discrete time returns that can only be applied to continuously compounded returns, in their specification of the market payoff in the "good" state, and in failing to take account of an illiquidity premium in corporate bond yields. Finally, their use of a cost of equity that is conditional upon no default occurring is likely to produce output prices that are too high relative to the NPV = 0 test. These features can be corrected, apart from the highly unconventional use of state prices with the market portfolio as the 'underlying' asset, and the failure to satisfy the NPV = 0 test. The latter failing is decisive.

Secondly, SFG's approach is very sensitive to estimates of several unobservable parameters, most particularly the market standard deviation, the recovery rate on defaulting bonds, the range in the firm's payoff from the best to worst market states sans default, and the expected default rate. These sensitivities must be compared with those from the CAPM, whose estimate for the cost of equity is sensitive to only estimates for the MRP and the equity beta. Prima facie, with twice as many parameters to estimate, SFG's approach seems much more sensitive to errors. Furthermore, there is a considerable body of empirical literature on estimating the CAPM parameters, and therefore considerable evidence about the extent of possible errors from its use (in the form of standard errors on the estimates of the MRP and beta). By contrast, there is much less evidence on the extent of estimation error in most of the parameters used in SFG's approach, most particularly the recovery rate in default for GGP bonds, the expected default rate on existing GGP bonds, and the range in the firm's payoff from the best to worst market states sans default. So, SFG's approach would seem to be more sensitive to estimation error and there is considerably less evidence about possible estimation errors. On this basis alone, I do not consider that it is a viable approach.

Thirdly, SFG's specification of the "good" and "bad" market states is incapable of reproducing the empirical estimate of the market standard deviation, and therefore cannot converge to any continuous time model of asset returns, including that assumed by Black and Scholes (geometric brownian motion for share returns).

Fourthly, SFG's claim that the standard PTRM assumes that default does not occur (and therefore that the allowed revenues are too low) is not correct. The standard PTRM cash flow modelling might fail to consider the full range of extreme events and the effect of this might be to produce allowed revenues or prices that are inadequate. However the cost of capital estimates used by regulators clearly allow for the possibility of default. In particular, the cost of debt used is a promised yield to maturity, which reflects the possibility of default, and the cost of equity estimated from the CAPM embodies a risk measure (beta) that reflects the market's (rather than the regulator's) perception of (systematic) risk and can therefore be reasonably presumed to reflect default risk to the extent it affects systematic risk. In fact, the use of the promised yield on debt will over compensate investors because the promised yield incorporates allowance for the default option held by equity investors, and the inclusion of this in the cost of debt used by regulators is unwarranted (because it is a mere transfer between debt holders and equity holders and therefore does not affect the appropriate regulatory revenues). So, even if the regulator fails to adequately allow for the possibility of extreme events and this gives rise to output prices that are too low, the over allowance in the cost of debt may more than compensate for it. Accordingly there are no strong grounds to suppose that the allowed revenues or prices are too low, as suggested by SFG.

1. Introduction

The ERAWA is currently assessing a regulatory proposal for a five year access arrangement from Goldfields Gas Transmission (GGT), relating to the Goldfields Gas Pipeline (GGP). In support of this, GGT has submitted a report by SFG (2014) arguing that GGP's systematic risk is higher than typical pipeline businesses in Australia, that the comparators used by ERAWA for estimating GGP's beta are unsuitable, and therefore that a different approach is required. This approach involves the use of a binomial option pricing framework and provides an estimated cost of equity for GGP conditional upon no default occurring because this is appropriate for regulatory purposes. In response, the ERAWA has raised the following questions with me:

- Is SFG's method theoretically sound for the purpose of estimating the cost of equity for GGP over the five-year regulatory period?
- Does the binomial approach converge to the continuous time approach of Black and Scholes?
- Are the parameter values in SFG's approach capable of being estimated to a degree of accuracy that is sufficient for the intended purpose?
- Is it true that regulators estimate the cost of capital conditional upon no default occurring, and therefore is it appropriate to estimate the cost of equity sans default for GGP?

This paper seeks to address these questions. I commence by describing and then assessing the theoretical merits of SFG's approach.

2. Theoretical Issues

2.1 SFG's Methodology

SFG (2014) obtains its estimate of the cost of equity for GGP through a multi-stage process. The analysis is premised upon a risk-free rate of 3.87%, an MRP of 6.67% (and hence an expected market return of 10.54%), and a promised yield on debt of 6.23%. To facilitate the explanation, I focus firstly upon their initial model in which there are only two possible outcomes on the market portfolio over the five year regulatory period. The process is as follows.

Firstly, SFG consider two possible outcomes on the market portfolio, with the first corresponding to the expected rate of return (10.54% per year and hence 65.03% over five years) plus one standard deviation (16.64% per year and hence 37.20% over five years), which totals 102.2% over five years and therefore a payoff of \$2.022 per \$1 invested. The other possible payoff is 1/\$2.022 = \$0.495. So, the "up" and "down" factors are U = 1.022 and D = 1/U = 0.495.

Secondly, SFG determine the probabilities of these two outcomes in order to satisfy the expected rate of return. Given that the expected return is 65.03% over five years, the probabilities must be 0.7565 for the "good" outcome and 0.2435 for the "bad" outcome:

$$\$1.6503 = 0.7565(\$2.022) + 0.2435(\$0.495)$$

Thirdly, SFG determine the risk-neutral probabilities for these two outcomes in order to satisfy the expected rate of return in a risk-neutral world (investors are risk neutral but all other features of the world are unchanged). In such a world the expected rate of return on all assets would be the risk-free rate (3.87% per year and hence 20.9% over five years). So, in order to produce an expected rate of return of 20.9% over five years, these risk-neutral probabilities must be 0.4677 and 0.5323:

$$\$1.209 = 0.4677(\$2.022) + 0.5323(\$0.495)$$

Fourthly, SFG consider the possible payoffs to debtholders, who receive a promised yield to maturity of 6.23% on an investment of \$60 per \$100 of RAB. If default does not occur, they will receive 6.23% per year on \$60, which is \$81.17. Default is assumed to be confined to the "bad" market situation, and default is assumed to result in a payoff of 43% of that owed (\$34.90).[1] Invoking the risk-neutral probabilities above implies that the expected outcome on the debt in the "bad" market return state must be \$64.96 as follows:

$$B = \$60 = \frac{0.4677(\$81.17) + 0.5323(\$64.96)}{1.209} \tag{1}$$

[1] Bankruptcy costs would drive down the recovery rate to the figure used of 43% and therefore do not require any separate accounting.

Fifthly, SFG derive the probabilities of not defaulting and defaulting conditional upon the "bad" market state occurring. The outcomes for debt holders in these states are $81.17 and $34.90 respectively and the expected outcome is $64.96 as derived above. So the probabilities of not defaulting and defaulting conditional upon the "bad" market state must be 0.6497 and 0.3503 respectively:

$$\$64.96 = 0.6497(\$81.17) + 0.3503(\$34.90)$$

So, the "good" market state, the "bad" market state with no default by GGP, and the "bad" market state with default by GGP have probabilities of 0.7565, 0.2435*0.6497 = 0.1582, and 0.2435*0.3503 = 0.0853 respectively. SFG note that the latter probability lies between the default rates on US Baa debt and Ba debt of 1.97% and 9.73%, using Moody's data from 1970-2013.

Sixthly, SFG determine the expected rate of return over five years on GGP's debt as 28.7% as follows:

$$E(R_d) = \left(\frac{\$81.17}{\$60} - 1\right)(0.7565 + 0.1582) + \left(\frac{\$34.90}{\$60} - 1\right)(0.0853) = 0.287$$

This expected return of 28.7% over five years implies an expected return of 5.18% per year and therefore a risk premium of 1.31% (relative to the risk-free rate) and a default premium of 1.05% (relative to the promised yield to maturity of 6.23%).

Seventhly, SFG derives the possible payoffs to the firm, per $100 of RAB. With default, the payoff to the firm must be that to debtholders ($34.90). In respect of the other two states, SFG assumes that the payoff in the "bad" market state with no default is 80% of that in the "good" market state (denoted *P*). This implies *P* = $153.68 and hence .8*P* = $122.94 so that

$$\$100 = \frac{\$153.68(0.4677) + [\$122.94(0.6497) + \$34.90(0.3503)](0.5323)}{1.209} \qquad (2)$$

Eighthly, SFG derive the payoffs to equity holders as the difference between that for the firm and that to the debtholders, being \$72.51 in the "good" market state, \$41.78 in the "bad" market state with no default and zero in the default state.

Ninthly, SFG derive the expected rate of return on the equity of the firm and the expected rate conditional on no default occurring as follows:

$$E(R_e) = \left(\frac{\$72.51}{\$40} - 1\right)(0.7565) + \left(\frac{\$41.78}{\$40} - 1\right)(0.2435)(0.6497) + (-1)(0.2435)(0.3503) = 0.5366$$

$$E(R_e | no\ default) = \frac{\left(\frac{\$72.51}{\$40} - 1\right)(0.7565) + \left(\frac{\$41.78}{\$40} - 1\right)(0.2435)(0.6497)}{0.7565 + (0.2435)(0.6497)} = 0.6799$$

These expected rates of return equate to 8.97% per year and 10.93% per year respectively. Consistent with standard regulatory practice of assuming no default, SFG (2014, para 123) considers that the more important number is 10.93% per year and is equivalent to an equity beta of 1.06 when the risk-free rate is 3.87% and the MRP is 6.67%:

$$.1093 = .0387 + (.1054 - .0387)(1.06)$$

This equity beta estimate of 1.06 is in excess of the estimate of 0.70 invoked by the ERAWA for a benchmark gas pipeline business (SFG, 2014, para 11). SFG extends this analysis to a multi-period scenario, involving the use of 60 monthly intervals in each of which there are two possible outcomes from the market portfolio, and concludes that the expected return on equity sans default is higher at 11.24%, which is equivalent to an equity beta of 1.10 (SFG, 2014, paras 18-19).

2.2 Analysis of SFG's Methodology

SFG's approach is radical in two ways. Firstly, it seeks to estimate the cost of equity for the GGP in a way that is consistent with the risk-free rate, the expected rate of return on the market portfolio, and the promised yield on debt. The conventional approach, using the CAPM, links the first three parameters but also requires an estimate of beta whilst the last parameter (the promised yield on debt) is linked to the risk-free rate but the DRP is estimated independently.

Thus, the conceptual advantage of SFG's approach is limited to ensuring consistency between the DRP estimate and the other parameters. Secondly, SFG's recommended cost of equity is the expected rate of return conditional upon default not occurring rather than being unconditional. SFG recognises that its approach is radical but argues that the components of it are "standard finance theory" (SFG, 2014, para 14). However, I do not consider that the last claim is correct and there are also a number of deficiencies in their methodology, as follows.

Firstly, option pricing analysis seeks to value an asset (the option) by reference to an underlying asset that determines the payoff on the option coupled with the rational exercise of a choice by the owner of the option. The seminal paper by Black and Scholes (1973) deals with European call and put options over shares, in which the underlying asset is the share and the owner of the option chooses whether or not to exercise it through payment of the exercise price. The principles have been applied to other financial assets including futures contracts (Black, 1976), foreign exchange (Grabbe, 1983), and bonds (Black et al, 1990). In addition, Brennan and Schwartz (1985) extend these principles to the value of a project involving natural resource extraction, in which the underlying asset is the natural resource and the firm has choices in respect of commencing, expanding, shutting down, and restarting production. Additional literature extends this framework to projects in general, with the underlying asset being the present value of the project benefits without the choices (McDonald and Siegel, 1986; Dixit and Pindyck, 1994; Trigeorgis, 1996). This is standard finance theory. However, in all of these cases, the underlying asset determines the payoff on the option coupled with the rational exercise of a choice by the owner of the option. By contrast, in SFG's analysis in which the underlying asset is the market portfolio and the other asset is the firm, the value of the market portfolio does not *determine* the payoff on the firm; it is merely correlated with the payoff on the firm and even includes the value of the firm itself. So, the outcomes on the market portfolio merely provide expected payoffs on the firm, around which there is considerable uncertainty. Furthermore, this linkage between the value of the two assets does not require the rational exercise of any choice by the firm. Accordingly, SFG's analysis is not option pricing analysis.

SFG (2014, para 42) claims that the underlying asset in their analysis is the firm value and the derivative asset is the value of equity but this is not the case; SFG invoke a binomial distribution for the returns on the market portfolio and therefore this portfolio is necessarily the underlying asset (as recognised by SFG in their para 14). SFG use the payoff on the firm to determine that on the equity of the firm, and the equity is a call option on the firm, but this is only secondary in their analysis. SFG (2014, para 42) also claim that their approach is an application of the methodology of Black and Scholes (1973) and Merton (1973). However, these papers deal with continuous time rather than binomial situations, they deal with the valuation of financial options (calls and puts) rather than companies, the underlying asset is a share price rather than the market portfolio, the underlying asset determines the value of the calls and puts at their maturity date rather than being merely correlated, and it does so in conjunction with the rational exercise of a choice by the owner of the option. So, SFG's analysis is not an application of Black and Scholes (1973) or Merton (1973). SFG do not refer to any other papers in the option pricing literature.

What SFG are using is in fact a "state pricing" approach, deriving from Arrow (1964) and Debreu (1959), with application to capital budgeting/firm valuation by Banz and Miller (1978) and Breeden and Litzenberger (1978). In this framework, one specifies expected outcomes for a firm or project conditional upon particular states of the market, and then values these conditionally expected payoffs using state prices (which differ from SFG's risk-neutral probabilities only by the risk-free rate).[2] This state pricing framework can be applied to situations in which the asset payoff is determined by an underlying asset, and therefore option pricing could be viewed as a special case of state pricing when the underlying asset determines the payoff on the asset of interest rather than being merely correlated with it. Since the special case does not hold here, SFG's analysis is therefore state pricing rather than option pricing. Within this state pricing framework, variations in outcomes around the expected payoffs on the firm for a given market state (good or bad) are treated as unpriced risk. However this state pricing approach to firm or project valuation is not "standard finance theory". Using SFG's (2014, para 276) own test for "standard finance theory" to be that "taught in undergraduate and master's finance courses", I have examined a collection of widely-used books in such courses: Grinblatt and Titman (2002), Brealey et al (2011), Damodaran (2011), Berk and DeMarzo (2014), Welch (2009), Ross et al (2013), and Copeland et al (2005). Of these books, only Copeland et al (2005, pp. 97-100) mentions the state pricing approach to firm or project valuation (briefly) and expresses doubts about its feasibility. SFG's addition of default and no default cases to each market outcome places them even further away from standard finance theory. Furthermore, SFG's paper does not contain even a single relevant reference to the

[2] SFG's entire analysis could have been conducted using state prices in substitution for risk-neutral probabilities, and therefore would not have required any reference to option pricing. See Appendix 1.

(limited) academic literature in support of such an approach. SFG have used state pricing, which is not standard finance theory, and confused it with option pricing, which is standard finance theory.

Secondly, all of the returns data used by SFG is discrete time data. However, SFG's formula for converting the standard deviation for annual returns (SD_1) into that for a period of T years (five years and one month in their case) is as follows:

$$SD_T = SD_1\sqrt{T} \tag{3}$$

and this formula is only valid if these standard deviations are over returns expressed in continuously compounded terms. Thus, SFG have confused the two types of returns. This can be remedied through the use of continuously compounded returns.

Thirdly, in using a binomial process, there are choices in the specification of the up and down factors (U and D), as noted by Jarrow and Turnbull (1996, section 4.4). However, SFG's approach does not correspond to any of those specifications. Furthermore, if the condition $D = 1/U$ is invoked (as SFG do) so as to reduce the number of branches beyond the one-period framework, then the value for U and the probability of its occurrence (q) must be as follows in order to ensure that the mean and variance of the binomial distribution converges on the empirical estimates as the two-outcome interval goes to zero:

$$U = e^{\sigma\sqrt{T}}, \quad q = 0.5\left[1 + \frac{\mu}{\sigma}\sqrt{T}\right] \tag{4}$$

where μ is the expectation of the continuously compounded annual rate of return, σ is its standard deviation, and T is the time interval (in years) over which the process yields only two outcomes (Cox et al, 1979). By contrast, letting $E(R_1)$ and SD_1 denote the expectation and standard deviation for the annual returns, SFG's value for U is as follows

$$U = \left[1 + E(R_1)\right]^T + SD_1\sqrt{T} \tag{5}$$

whilst the probability of this outcome is chosen by SFG to ensure that the expected rate of return from the two-outcome distribution matches the empirical estimate:

$$p = \frac{\left[1 + E(R_1)\right]^T - D}{U - D} \qquad (6)$$

So, in effect, SFG avoids any error in the mean at the potential expense of error in the standard deviation. To assess the extent of the error in the standard deviation, I consider the two-outcome interval of one month that is used by SFG in their multi-period analysis. In this case, following equation (5):

$$U = \left[1.1054\right]^{(1/12)} + (0.1489)\sqrt{1/12} = 1.0514$$

and therefore D = 0.9511. Following (6), the probability of the "good" state is p = 0.5713 so as to obtain the correct expectation of 1.0084. The standard deviation of this distribution of returns over a one month interval is then is as follows:

$$SD = \sqrt{(1.0514 - 1.0084)^2(0.5713) + (0.9511 - 1.0084)^2(0.4287)} = .050$$

So, the standard deviation of this distribution is .050 whilst the empirical estimate that underlies the calculations is .043, and therefore the error is an overstatement of 15%.[3] Furthermore, over a succession of periods, the error grows. For example, over a period of one year (involving 12 steps in the binomial process), and using these values for U and D, the resulting possible outcomes over the course of one year are as shown in the central column of Table 1 below. Using the value for p, of 0.5713, the resulting probabilities are shown in the last column. Deducting 1 from the outcomes to obtain the rate of return, the mean and standard deviation of this distribution are 0.1056 and 0.1898 respectively, and the latter deviates from the empirical value underlying the calculations (0.1489) by 23%. Interestingly, SFG (2014, para 39) claims that the returns and probabilities in their binomial framework need to be consistent with their empirical estimates of both the expected market return and standard deviation, but they have failed to do so in respect of the standard deviation. By contrast, application of equation (4)

[3] The fact that the estimate of .043 is incorrectly derived from that of the annual estimate by wrongly invoking equation (3) is not an impediment to conducting this test.

yields a binomial distribution over the course of a year whose mean is within 1%, and whose standard deviation is within 2.3%, of the empirical estimates underlying the calculations, and these details are shown in Appendix 2. So, given the choice of $D = 1/U$, SFG have erred in using equation (5) and (6) rather than (4). This can be remedied by using equation (4).

Table 1: Distribution of Returns from SFG's Approach

No. Down	Outcome	Probability
0	$(1.0514)^{12} = 1.8248$	$(0.5713)^{12} = 0.00121$
1	$(1.0514)^{11}(0.9511) = 1.6507$	$12(0.5713)^{11}(0.4287) = 0.01088$
…		
…		
11	$(1.0514)(0.9511)^{11} = 0.6057$	$12(0.5713)(0.4287)^{11} = 0.00062$
12	$(0.9511)^{12} = 0.5479$	$(0.4287)^{12} = 0.00004$

Fourthly, it is implicit in SFG's equation (1) above that the DRP (6.23% - 3.87%) is due entirely to the possibility of default. However there is a considerable body of literature on the DRP impact arising from the inferior liquidity of corporate bonds relative to the risk-free asset (government bonds), with Amihud et al (2005, section 3.3.2) providing a comprehensive survey. More recently, Almeida and Philippon (2007, Table II) summarise results from a number of papers, in which the proportion of the DRP due to default ranges from 34% to 71% for BBB bonds (and the rest due to illiquidity). Furthermore, like SFG, Almeida and Philippon sought to estimate the probability of default from the DRP but (unlike SFG) they deducted out an estimate of the illiquidity premium. Unsurprisingly in view of their failure to account for illiquidity, SFG (2014, page 13) obtain an estimate of the default probability from their analysis that is significantly more (over four times) than that of the average default rate in Moody's data for Baa bonds (8.53% in the analysis above and 9.65% in their multi-period extension, versus 1.97% in the Moody's data). Remarkably, SFG (2014, paras 62-63) seem to recognise that there is a problem here but brush it off, presumably because they did not appreciate that the discrepancy could be explained by an illiquidity premium. Equally remarkably, SFG (2014, para 77) critique the standard regulatory approach as potentially leading to inconsistencies between the observed cost of debt and the estimated cost of equity, but have committed a more egregious mistake themselves. Given that SFG invoke Moody's data to estimate the expected recovery rate in default (43%), this suggests choosing an expected default rate in their model equal to the average historical rate in the Moody's data (1.97%). Letting p_d denote the probability of default conditional upon the "bad" market state occurring, and recalling that the probability of the "bad" market state is 0.2435, it follows that

$$0.2435 p_d = .0197$$

and therefore p_d = 0.0809. Consequently the expected payoff to debtholders in the "bad" market state would be

$$E(Debt\ Payoff \,|\, "bad"\ state) = \$81.17(0.9191) + \$34.90(0.0809) = \$77.43 \qquad (7)$$

Letting Z denote the illiquidity allowance per year in rate of return terms, the debt value of \$60 would then satisfy the following equation:[4]

$$\$60 = \frac{\$81.17(0.4677) + \$77.43(0.5323)}{(1 + .0387 + Z)^5} \qquad (8)$$

The solution is Z = .0183, and therefore 77% of the DRP (.0623 - .0387 = .0236) would be an allowance for illiquidity (with rest allowing for the possibility of default). Turning now to equity holders, and letting Q denote the payoff to the firm in the "good" market state, the value for p_d derived above (0.0809) implies that the probability of no default in the "bad" market state is 0.9191 and therefore that the value of equity (\$40) must satisfy the following equation:

$$\$40 = \frac{(Q - \$81.17)(0.4677) + (.8Q - \$81.17)(0.5323)(0.919)}{1.209} \qquad (9)$$

The solution is Q = \$146.71. So, the payoff to equity would be \$146.71 - \$81.17 = \$65.54 in the "good" market state and .8(\$146.71) - \$81.17 = \$36.20 in the "bad" market state without default. Accordingly the expected rate of return on the equity of the firm and the expected rate conditional on no default occurring would be as follows:

[4] This differs from SFG's equation (1) only in adding the illiquidity premium to the risk-free rate, to obtain the 'risk-free rate for corporate bonds'.

$$E(R_e) = \left(\frac{\$65.54}{\$40} - 1\right)(0.7565) + \left(\frac{\$36.20}{\$40} - 1\right)(0.2435)(0.9191) + (-1)(0.2435)(0.0809) = 0.4423$$

$$E(R_e | no\ default) = \frac{\left(\frac{\$65.54}{\$40} - 1\right)(0.7565) + \left(\frac{\$36.20}{\$40} - 1\right)(0.2435)(0.9191)}{0.7565 + (0.2435)(0.9191)} = 0.4713 \quad (10)$$

These expected rates of return equate to 7.60% and 8.03% per year respectively. Not only are both rates significantly less than SFG's results (8.97% and 10.93%) but the difference between these two rates of 0.43% is only 20% of that obtained by SFG (10.93% - 8.97% = 1.96%) merely through recognising the existence of an illiquidity premium in corporate bonds. Furthermore the beta estimate that would have yielded an expected return of 8.03% would have been 0.62, which is now below the ERAWA's estimate of 0.70. So, this allowance for the illiquidity premium completely overturns SFG's conclusion that a beta of 0.70 is too low for GGP. This deficiency in SFG's approach can be remedied, by simply allowing for an illiquidity premium, but it will add to the number of parameters that require estimation and therefore add to the potential for error in SFG's approach.

Fifthly, even within SFG's multi-period analysis, all payoffs are assumed to occur in five years and therefore firms retain all cash flows from operations over the course of five years (rather than paying dividends) and debtholders do not receive any interest for five years. This is well outside the bounds of standard financial analysis, which assumes payment intervals no less frequently than annual. It is also far removed from the reality of business operations and is likely to have affected SFG's estimate of the cost of equity. To illustrate the problem, the deferral of interest payments to debtholders for five years will magnify the significance of any defaults and introduces a disparity between their model (which assumes payouts every five years) and the empirical data on default rates and recovery rates (which arise in a world of interest payments that are made on an annual or more frequent basis). By contrast, the PTRM assumes that cash flows arise annually and interest payments are also made at that frequency. This shortcoming in SFG's work could be remedied but only at the price of significantly increasing the complexity of their analysis.

Sixthly, a fundamental test that any approach to setting regulatory prices must face is the NPV = 0 principle; expected revenues must be such that their present value net of opex and capex must equal the initial investment. However SFG does not explicitly consider the issue. SFG (2014, para 85, para 123) claims that regulatory prices are set assuming that default will not occur, and therefore should be based upon an expected rate of return on equity that is also conditional upon there being no default (10.93%). They also imply that the cost of debt that should be used is the promised yield (6.23%) but they do not assess whether these choices would satisfy the NPV = 0 test. Over the five-year period, this would produce a 'WACC' of

$$'WACC' = (1.1093)^5(0.4) + (1.0623)^5(0.6) - 1 = 0.4836 \tag{11}$$

The regulator would then apply this 'WACC' to the RAB of $100 to produce expected cash flows of $148.36. To convert this into an output price, I will assume that the possible outcomes presented in section 2.1 of $153.68, $122.94 and $34.90 (with probabilities of 0.7565, 0.1582, and 0.0853 respectively) reflect output levels of 153.68, 122.94, and 34.90 units respectively with an output price of $1 (with no opex). If the regulator fails to recognise the scenario that gives rise to the default outcome, they would recognise possible output levels of only 153.68 and 122.94 units, with perceived probabilities of 0.7565/0.9147 = 0.827 and 0.1582/0.9147 = 0.173. So, they would perceive an expected output level of 148.36 units. Accordingly, they would set an output price of $148.36/148.36 = $1. This produces possible cash flows of $153.68, $122.94 and $34.90 with probabilities of 0.7565, 0.1582, and 0.0853 as before, with an expectation of $138.70. To value this, the correct WACC to apply is the unconditional cost of equity (8.97% per year) and the unconditional expected rate of return on debt (5.18%), to yield a present value equal to the RAB of $100 as follows:[5]

$$V = \frac{\$138.70}{(1.0897)^5(0.4) + (1.0518)^5(0.6)} = \$100$$

If this is SFG's intention concerning regulatory behaviour, it would seem to satisfy the NPV = 0 test. Thus, SFG favours use of an increased 'cost of equity' coupled with the promised yield on debt to offset the assumed failure by regulators to recognise the default scenario in their estimate of the expected output level. However, all of this is premised upon regulators forming an expectation about future outcomes in which the extreme cases that lead to default are

[5] The appropriate cost of debt to use here is the promised yield less the allowance for the default option possessed by equity holders, yielding 5.18%, because this default option is a mere transfer between the two suppliers of capital rather than a cost for suppliers in aggregate. Section 4 elaborates upon this matter.

disregarded, and even SFG (2014, paras 23-24) is not confident about this hypothesis because they refer to it only after first asserting that regulators consider only the "most likely case". Furthermore, this hypothesis that regulators overlook the default scenarios is a very strong assumption. If regulators appreciate the full distribution of possible output levels, with an expected output of 138.70 units, their use of the 'WACC' favoured by SFG (48.36% over five years) would lead to them setting an output price of $1.07 as follows:

$$S = \frac{\$100(1.4836)}{138.70} = \$1.07$$

Since the correct output price is $1, this output price of $1.07 will be too high by 7%. Alternatively, if regulators use some sort of typical output level (as SFG claims that they do), this typical level must be less than the expectation conditional on no default (148.36 units), the output price set by them will still then be above $1 and therefore will still be too high. So, if SFG's approach to setting WACC were adopted, it would most likely lead to an output price that was too high. Even SFG implicitly accepts this because they favour equation (11) for setting the WACC whilst also believing that regulators estimate the expected output from the 'typical' value, and this combination will yield output prices that are too high.

In summary, SFG's theoretical analysis significantly diverges from standard finance theory in using state prices with the market portfolio as the 'underlying' asset to the value of a firm, and also in assuming payoffs from regulated assets only at five-yearly frequencies. In addition, their analysis is wrong in applying a formula to discrete time returns that can only be applied to continuously compounded returns, in their specification of the market payoff in the "good" state, and in failing to take account of an illiquidity premium in corporate bond yields. Finally, their use of a cost of equity that is conditional upon no default occurring is likely to produce output prices that are too high relative to the NPV = 0 test. These features can be corrected, apart from the highly unconventional use of state prices with the market portfolio as the 'underlying' asset, and the failure to satisfy the NPV = 0 test. The latter failing is decisive.

3. Sensitivity to Parameter Estimates

SFG (2014, para 175) conduct sensitivity analysis on the parameter estimates adopted by them. Within the multi-period case, inter alia, they estimate the sensitivity of the expected return on equity sans default to changes in the market standard deviation, the recovery rate on defaulting bonds, and the range in the firm's payoff from the best to worst market states sans default.

In respect of the market standard deviation, SFG (2014, page 7) estimate this at 16.64% per year based upon Australian market returns from 1883-2013, and then they reduce it to 14.89% for reasons of presentational convenience (SFG, 2014, para 127). They then show that a 1% change in the estimate changes the expected rate of return on equity sans default by 0.23%. Regardless of which estimate for this parameter is used, the process of estimating it raises the question of its statistical reliability. A possible response to this would be to argue that estimating it from historical returns data is comparable to estimating the MRP on the Australian market from the same period. However, I am not aware of any regulator who does so; all of them estimate the MRP from a variety of sources so as to improve the reliability of the estimate. An alternative approach to estimating the market volatility over five years is the volatility implicit in the prices of options written on the market index ("implied volatilities"), for which there is a considerable academic literature (Hull, 1997, section 11.10). SFG do not refer to this.

In respect of the recovery rate, SFG (2014) estimate this at 43% based on Moody's data, note that the recovery rates are very similar for both Baa and Ba bonds, and that a 10% change (to 33% or 53%) would change the expected rate of return on equity sans default by 0.70% (SFG, 2014, para 175). SFG's reference to similar default rates on these two categories of bonds suggests that the estimate is reliable. However, within each such category, there will be wide variation in recovery rates across firms depending upon the alternative uses for the assets and the scenarios inducing default. For example, if defaults within a sector are typically induced by events that undermine the viability of all such businesses and the assets have no alternative uses, the expected recovery rate for debtholders will be close to zero. By contrast, if defaults within a sector are typically induced by poor management within individual firms, default will typically lead to new management rather than the liquidation of the business, and therefore the expected recovery rate for debtholders will be high. Alternatively, if defaults within a sector are typically induced by events that undermine the viability of all such businesses but the assets of these businesses are largely tangible and have alternative uses, default will typically lead to the collapse of the businesses but the expected recovery rate for debtholders will still be high. The situation regarding GGP is clearly not typical of businesses because there is no competition.

Thus, if default occurs, it will most likely be because the business is no longer viable. Furthermore, the assets have no alternative uses. So, if default occurs, the recovery rate for debtholders is likely to be unusually low. As shown by SFG, lower recovery rates for a given cost of debt imply a lower probability of bankruptcy and therefore a lower cost of equity. Thus, not only is there considerable uncertainty about the appropriate recovery rate in default for GGP and therefore considerable uncertainty about the cost of equity when using SFG's approach but SFG's use of the market average recovery rate is likely to have overestimated the recovery rate for GGF and therefore overestimated its cost of equity.

In respect of the range in the firm's payoff from the best to worst market states sans default, SFG (2014, para 158) assume that the firm's payoff in the top 8.5% of market outcomes over five years is 15% larger than for the 'typical' market outcome (realised market return is equal to the expectation), that the firm's payoff in the bottom 6.69% of market outcomes is 15% less than that for the 'typical' case, and that the adjustment % varies within this band of ±15% for the remaining cases in accordance with the probability of the outcome relative to that of the 'typical' outcome. The range is then from 0.85 to 1.15. SFG (2014, para 175) consider the effects of widening or narrowing this band on the estimate for the expected rate of return on equity sans default. In particular, varying the band by ±0.10 (to .80 – 1.20 or to 0.90- -1.10) changes the expected rate of return on equity sans default by 1%. However, unlike the expected recovery rate or the standard deviation of market returns, there is no empirical evidence on the appropriate width of the band. SFG (2014, section 3) determine various combinations of volume reductions and shortfalls in the capacity payments that are required from customers that are consistent with an outcome that is 15% less than 'typical', and these possibilities include shortfalls in both volume and capacity payments of 6.21%. SFG concludes that these reductions are not very substantial, goes on to highlight much more extreme possibilities arising from some customers ceasing operations (SFG, 2014, paras 257-266), but then concludes in para 272 that the ±15% band is appropriate. However, nothing in this analysis supports the use of the 15% band chosen by them, as opposed to (say) 12% or 20%. The 15% band has simply been 'plucked out of the air'. Furthermore, this band is conceptually similar to the equity beta within the CAPM. SFG is critical of the estimate adopted by the ERAWA but their alternative lacks even the empirical exercise underlying the ERAWA's choice of the beta estimate.

In addition to these parameters, and following the discussion in section 2.2 above, it would also be necessary to estimate the probability of default on GGP's bonds. SFG cites the historical default rate reported by Moody's for bonds of the relevant credit rating (Baa) but this data is averaged over a considerable period and therefore at best represents an expected outcome over the full set of future economic conditions. By contrast, the desired default probability for the present regulatory purposes is that implicit in GGP's current cost of debt and this may diverge significantly from the Moody's average. Furthermore, the Moody's data invoked by SFG averages over all firms with the same credit rating, and its use therefore presumes that regulated utilities would experience the same default rates as firms in general with the same credit rating. However, PwC (2012, Table 7) reveals that the default rates reported by Moody's for regulated utilities are markedly less than firms in general with the same credit rating. This raises the interesting question of the extent to which this disparity is chance or not, and therefore whether to use the broader Moody's data or just that for regulated utilities. A possible approach here would be to estimate the illiquidity premium reflected in the current cost of debt, and then deduce the current default probability in the "bad" market state from this. In particular, upon estimating the current illiquidity premium Z, substitution into equation (8) would yield the expected payoff to debt holders in the "bad" market state, substitution of this into equation (7) would yield the default probability conditional upon the "bad" market state, substitution of this into equation (9) would yield the firm's payoff in the "good" market state (Q), and substitution of this into equation (10) would yield the expected rate of return on equity sans default.

However, as revealed by Almeida and Philippon (2007, Table II), the range of estimates for the proportion of the DRP attributable to default is high and therefore so too would the part due to illiquidity. Furthermore, estimates of the allowance for illiquidity can vary quite substantially over time; for example, Dick-Nielsen et al (2012, Table 4) shows that the allowance during the 2007-2009 period was 20 times that during the 2005-2007 period for Baa bonds with 5-30 year terms to maturity. All of this implies that any estimate for the illiquidity premium on existing Baa bonds would be subject to considerable estimation error and therefore aggravate the existing such problems in SFG's approach. To determine the impact on SFG expected return on equity sans default, an estimate of the illiquidity premium consistent with the current cost of debt in SFG's analysis (6.23%) and the historical average default rate on Baa bonds over five years (1.97%) is 1.83%, and this yields an estimate for the expected rate of return on equity sans default of 8.03% as shown in section 2.2. If this estimate for the illiquidity premium is reduced by 0.50% to 1.33%, substitution into equation (8) would yield the expected payoff to debt holders in the "bad" market state of $73.92, substitution of this into equation (7) would yield the default probability conditional upon the "bad" market state of 0.1568, substitution of this into equation (9) would yield the firm's payoff in the "good" market state (Q) of $148.47, and substitution of this into equation (10) would yield the expected rate of return on equity sans default of 8.80%. So, reducing the estimate of the liquidity premium by 0.50% raises the expected rate of return on equity sans equity by 0.77%. So, the expected rate of return on equity sans default is very sensitive to the illiquidity premium.

In summary, SFG's approach is very sensitive to estimates of several unobservable parameters, most particularly the market standard deviation, the recovery rate on defaulting bonds, the range in the firm's payoff from the best to worst market states sans default, and the expected default rate. These sensitivities must be compared with those from the CAPM, whose estimate for the cost of equity is sensitive to only estimates for the MRP and the equity beta. Prima facie, with twice as many parameters to estimate, SFG's approach seems much more sensitive to errors. Furthermore, there is a considerable body of empirical literature on estimating the CAPM parameters, and therefore considerable evidence about the extent of possible errors from its use (in the form of standard errors on the estimates of the MRP and beta). By contrast, there is much less evidence on the extent of estimation error in most of the parameters used in SFG's approach, most particularly the recovery rate in default for GGP bonds, the expected default rate on existing GGP bonds, and the range in the firm's payoff in the best to worst market states sans default. So, SFG's approach would seem to be more sensitive to estimation error and there is considerably less evidence about possible estimation errors. On this basis alone, I do not consider that it is a viable approach.

4. The Relevance of Default to Regulatory Practice

The analysis so far concludes that SFG's methodology is theoretically deficient and too sensitive to the values of several unobservable parameters to be viable even if there were no theoretical limitations. Nevertheless, it is still necessary to assess SFG's (2014, section 2.1) argument that the standard PTRM assumes that default does not occur and therefore that the allowed revenues are too low. In support of this claim, SFG refers to payments to debtholders and tax payments that presume no default. However, the interest payments to debtholders that are invoked under the standard PTRM are the promised payments, which incorporate an allowance for the possibility of default, and therefore assuming that such payments will always be made *raises* the allowed revenues and therefore could not cause the allowed revenues to be too low.[6] SFG also refers to regulators considering only a typical case rather than taking an expectation over the distribution of all possible outcomes. However, this could go in either direction and will still be correct if the atypical events are symmetrically distributed around that typical outcome. Furthermore, even if the downside outweighs the upside, at least some of these downside events would lead regulators to provide ex-post compensation to firms and this might compensate for the asymmetry. SFG also refers to the possibility of regulators forming an expectation over the distribution of all possible outcomes excluding default, but this is a very strong assumption about regulatory behaviour and SFG do not present any evidence in support of it. By contrast, the cost of capital estimates used by regulators clearly allow for the possibility of default. In particular, the cost of equity is estimated from the Officer (1994) version of the CAPM and, like all versions of the CAPM, its risk measure (beta) reflects the market's (rather than the regulator's) perception of (systematic) risk and can therefore be reasonably presumed to reflect default risk to the extent it affects systematic risk. Furthermore the cost of debt used is a promised yield to maturity and this reflects the possibility of default, i.e., it is higher to reflect that possibility. In fact, since the promised yield to maturity comprises the expected return to bondholders plus an allowance for bankruptcy costs plus an allowance for the value of the default option possessed by equity holders, the inclusion of the latter in the cost of debt used by regulators leads to over compensation for firms (because it is a mere transfer between debt holders and equity holders and therefore does not affect the appropriate regulatory revenues). So, even if the regulator fails to adequately allow for the possibility of extreme events and this gives rise to output prices that are too low, the over allowance in the cost of debt may more than compensate for it.

To illustrate these points, suppose that an unlevered firm acquires assets now for $100m that will deliver a payoff of $55m, $105m or $155m in one year with probabilities of 20%, 60%, and 20% respectively, investors are risk neutral, the risk free rate is 5%, and there is no opex or taxes (personal or corporate).[7] The expected payoff is then $105m, the cost of capital would be the unlevered cost of equity, which would equal the risk-free rate of 5%, and therefore the value of the firm would be $100m, which equals the purchase price of the assets. Suppose now

[6] Determining the taxes to be paid using the same 'highest level' interest payments yields taxes that are at the 'lowest level', which exerts a downward effect on allowed revenues, but the net effect on allowed revenues is still upwards because each $1 of interest reduces taxes by less than $1.

[7] The example is intended only to illustrate the principle and not also the scale of the effect.

that the firm acquires some debt finance, promises a payment of $60m to debt holders (principal plus interest), and there are no bankruptcy costs, i.e., even in the presence of debt, the possible payoffs from the firm in one year are still $55m, $105m, or $155m with probabilities of 20%, 60% and 20% respectively. So, the value of the firm is still $100m. However, given the default option possessed by equity holders, the payoff on the debt will be $60m in two states and only $55m in one state. Assuming (so as to focus on the key point) that there is no illiquidity premium on debt, the value now of the debt will be $56.19m as follows:

$$B = \frac{.20(\$55m) + .80(\$60m)}{1.05} = \$56.19m$$

So, a promise of $60m will allow the firm to borrow $56.19m, and the promised yield on debt will then be 6.78%, comprising the risk free rate of 5% and compensation of 1.78% to debt holders for expected default losses, which arise purely from the default option possessed by equity holders rather than from bankruptcy costs. Since the debt comprises 56.19% of firm value then the WACC defined using the promised yield on debt as the cost of debt will be

$$WACC = .4381ke + .5619kd = .4381(5\%) + .5619(6.78\%) = 6.0\%$$

If the business were regulated and the regulator allowed 6.0% on the firm's asset base of $100m, the regulator would then set a price or revenue cap so that the firms' expected payoffs in one year would be $106m.

The regulatory arrangements for the GGP are essentially those of a price cap (a price cap with limited scope for adjustment), but I consider both revenue and price caps because SFG's approach could be more generally employed. I start with a price cap. If the regulator correctly estimates expected sales at 105m (arising from possible sales of 55m, 105m, or 155m with probabilities of .20, .60, and .20), a price cap of $1.01 would then be applied, leading to expected revenues of $106m. The resulting value now of the firm would then be $101m as follows:

$$V_0 = \frac{\$106m}{1.05} = \$101m \qquad (12)$$

Since the purchase price of the assets is only $100m then the shareholders would have been gifted $1m through the regulator defining the cost of debt as the promised yield. By contrast, if the regulator excludes the default option component of the cost of debt, and therefore sets the cost of debt at 5%, and therefore the WACC also at 5%, this would lead to expected revenues of $105m and therefore a price cap of $1. The resulting value now of the firm would be $100m as follows:

$$V_0 = \frac{\$105m}{1.05} = \$100m$$

This matches the initial investment. So, in this case, the appropriate regulatory policy is to exclude the default option from the cost of debt. Nothing in this process is inconsistent with the default possibility; the business bears the volume risk under a price cap and therefore, if sales are 55m, the revenues of the business will not cover its interest costs and default will occur. Naturally, the regulator might not have correctly estimated the expected sales at 105m, but any such error could be in either direction. SFG envisages the situation in which the regulator perceives the expected sales to be 122m, because they fail to recognise the worst possible state in forming that expectation. Accordingly, they will set the price cap at $106m/122m = 0.87 and therefore this will be too low. This is a rather improbable scenario.

The situation under a revenue cap is more complex. If the regulator uses the promised yield on debt and therefore estimates the WACC at 6%, their revenue target would be $106m. With expected sales correctly estimated at 105m, the output price would be set at $1.01 as above, reduced to $0.68 is sales were 155m, and raised to $1.93 if sales were 55m so as to meet the revenue target. So long as consumers will pay the $1.93 in the latter scenario, default would now be impossible and therefore a cost of debt of 6.78% would not arise. Thus, internal consistency in the example would require that default be possible and therefore that consumers would not purchase the 55m units at $1.93. For example, suppose that the price were not raised above $1.01 for this very reason. So, revenues would be $106m with probability 0.8 and $55.6m with probability 0.2. So, the expected revenues would actually be $95.9m, and the value of the business would be $91.4m as follows:

$$V_0 = \frac{\$95.9m}{1.05} = \$91.4m$$

Alternatively, if the regulator uses the cost of debt exclusive of the default option allowance, the value will be even lower. So, the regulator’s use of the promised yield on debt only mitigates the inadequacy of the regulatory allowance that arises from consumers being unwilling to pay a sufficiently higher output price when demand collapses to permit the revenue target to be met. Thus the regulator fails to appreciate that the revenue target cannot be attained in extremely low demand scenarios, and therefore sets an initial output price ($1.01) that is too low. This scenario is consistent with SFG's claims, even though they do not explicitly refer to it.

The situation just examined presumes that regulated businesses are stand-alone businesses, and this is typically not the case. In this case, the regulated business may be free of any risk (because the revenues will always be achieved) but the DRP may reflect default risk due to the other activities of the firm. Accordingly, regulatory use of the promised yield on debt will lead to excess revenues being allowed. For example, suppose the promised yield on the firm's debt is 6% and therefore the regulator allows regulatory revenues of $106m. If the regulated business is free of risk, the present value of the business will be $101m as in equation (12), and therefore the allowed revenues would be too high.

In summary, SFG's claim that the standard PTRM assumes that default does not occur (and therefore that the allowed revenues are too low) is not correct. The standard PTRM cash flow modelling might fail to consider the full range of extreme events (or their consequences) and the effect of this might be to produce allowed revenues or prices that are inadequate. However the cost of capital estimates used by regulators clearly allow for the possibility of default. In particular, the cost of debt used is a promised yield to maturity, which reflects the possibility of default, and the cost of equity estimated from the CAPM embodies a risk measure (beta) that reflects the market's (rather than the regulator's) perception of (systematic) risk and can therefore be reasonably presumed to reflect default risk to the extent it affects systematic risk. In fact, the use of the promised yield on debt will over compensate investors because the promised yield incorporates allowance for the default option held by equity investors, and the inclusion of this in the cost of debt used by regulators is unwarranted (because it is a mere transfer between debt holders and equity holders and therefore does not affect the appropriate regulatory revenues). So, even if the regulator fails to adequately allow for the possibility of extreme events and this gives rise to output prices that are too low, the over allowance in the cost of debt may more than compensate for it. Accordingly there are no strong grounds to suppose that the allowed revenues or prices are too low, as suggested by SFG.

5. Conclusions

This paper has addressed a number of issues raised by the ERAWA and the conclusions are as follows.

Firstly, SFG's theoretical analysis significantly diverges from standard finance theory in using state prices with the market portfolio as the 'underlying' asset to the value of a firm, and also in assuming payoffs from regulated assets only at five-yearly frequencies. In addition, their analysis is wrong in applying a formula to discrete time returns that can only be applied to continuously compounded returns, in their specification of the market payoff in the "good" state, and in failing to take account of an illiquidity premium in corporate bond yields. Finally, their use of a cost of equity that is conditional upon no default occurring is likely to produce output prices that are too high relative to the NPV = 0 test. These features can be corrected, apart from the highly unconventional use of state prices with the market portfolio as the 'underlying' asset, and the failure to satisfy the NPV = 0 test. The latter failing is decisive.

Secondly, SFG's approach is very sensitive to estimates of several unobservable parameters, most particularly the market standard deviation, the recovery rate on defaulting bonds, the range in the firm's payoff from the best to worst market states sans default, and the expected default rate. These sensitivities must be compared with those from the CAPM, whose estimate for the cost of equity is sensitive to only estimates for the MRP and the equity beta. Prima facie, with twice as many parameters to estimate, SFG's approach seems much more sensitive to errors. Furthermore, there is a considerable body of empirical literature on estimating the CAPM parameters, and therefore considerable evidence about the extent of possible errors from its use (in the form of standard errors on the estimates of the MRP and beta). By contrast, there is much less evidence on the extent of estimation error in most of the parameters used in SFG's approach, most particularly the recovery rate in default for GGP bonds, the expected default rate on existing GGP bonds, and the range in the firm's payoff from the best to worst market states sans default. So, SFG's approach would seem to be more sensitive to estimation error and there is considerably less evidence about possible estimation errors. On this basis alone, I do not consider that it is a viable approach.

Thirdly, SFG's specification of the "good" and "bad" market states is incapable of reproducing the empirical estimate of the market standard deviation, and therefore cannot converge to any continuous time model of asset returns, including that assumed by Black and Scholes (geometric brownian motion for share returns).

Fourthly, SFG's claim that the standard PTRM assumes that default does not occur (and therefore that the allowed revenues are too low) is not correct. The standard PTRM cash flow modelling might fail to consider the full range of extreme events and the effect of this might be to produce allowed revenues or prices that are inadequate. However the cost of capital estimates used by regulators clearly allow for the possibility of default. In particular, the cost of debt used is a promised yield to maturity, which reflects the possibility of default, and the cost of equity estimated from the CAPM embodies a risk measure (beta) that reflects the market's (rather than the regulator's) perception of (systematic) risk and can therefore be reasonably presumed to reflect default risk to the extent it affects systematic risk. In fact, the use of the promised yield on debt will over compensate investors because the promised yield incorporates allowance for the default option held by equity investors, and the inclusion of this in the cost of debt used by regulators is unwarranted (because it is a mere transfer between debt holders and equity holders and therefore does not affect the appropriate regulatory revenues). So, even if the regulator fails to adequately allow for the possibility of extreme events and this gives rise to output prices that are too low, the over allowance in the cost of debt may more than compensate for it. Accordingly there are no strong grounds to suppose that the allowed revenues or prices are too low, as suggested by SFG.

APPENDIX 1: State Pricing

I consider the scenario in which the market portfolio has only two possible outcomes over a five-year period ($2.022 and $0.495 per $1 of current market value). Let V_1 denote the value now of an asset that pays $1 if the market portfolio outcome is "good" and zero otherwise, and V_2 denote the value now of an asset that pays $1 if the market portfolio outcome is "bad" and zero otherwise. Each $1 that the market portfolio pays off in the "good" state is worth V_1 today, and each $1 that the market portfolio pays off in the "bad" state is worth V_2 today. So, the value now of the market portfolio per $1 of that value can be expressed as the following linear function of its possible payoffs and the state prices:

$$\$1 = \$2.022 V_1 + \$0.495 V_2$$

The same approach can be applied to the riskless asset, delivering a payoff of $1.209 per $1 of current value in both future states:

$$\$1 = \$1.209 V_1 + \$1.209 V_2$$

Solving the last two equations yields V_1 = $0.3868 and V_2 = $0.4403.[8] These state prices could be used to determine the expected payoff to debtholders in the "bad" state (*W*), per $60 of current value, as follows:

$$\$60 = \$81.17(0.3868) + W(0.4403)$$

and the solution is *W* = $64.96 as in equation (1). Given this, and the payoffs to debtholders under no default ($81.17) and default ($34.90), the probability of no default conditional upon the "bad" state arising is 0.6497 as before. In addition, these state prices could also be used to determine the expected payoff to the firm in the "good" market state (*P*), per $100 of current value, as follows:

$$\$100 = P(0.3868) + [.8P(0.6497) + \$34.90(0.3503)](0.4403)$$

[8] Multiplying the state prices by 1.209 yields 0.4677 and 0.5323 for states 1 and 2 respectively, and these are the "risk neutral probabilities".

and the solution would is P = \$153.68, as in equation (2). The expected rates of return on equity can then be determined as shown in section 2.1. Thus, the use of state prices substitutes for the use of risk-neutral probabilities and therefore there would be no cause for referring to option pricing (with which risk-neutral probabilities are associated).

APPENDIX 2: The Accuracy of Equation (4)

This Appendix tests the accuracy of equation (4) over the course of a year. To do so, the first step is to specify empirical values for parameters μ and σ within these equations, which are defined as the expectation and standard deviation of the continuously compounded rate of return. I choose values of μ = 0.08895 and σ = 0.15, so as to closely match the discrete time return counterparts invoked by SFG. To see this, letting R denote the discrete time rate of return over one year, Z the standard normal random variable, and assuming that the continuously compounded return is normally distributed, it follows that

$$1+R=e^{\mu+\sigma Z}$$

So

$$E(1+R)=e^{\mu+.5\sigma^2}$$

Substitution of the values for μ = 0.08895 and σ = 0.15 into the last equation implies that $E(R)$ = 0.1054, which matches SFG's empirical estimate. In addition

$$\begin{aligned} VAR(1+R) &= VAR(e^{\mu+\sigma Z}) \\ &= E\left[(e^{\mu+\sigma Z})^2\right]-\left[E(e^{\mu+\sigma Z})\right]^2 \\ &= E\left[e^{2\mu+2\sigma Z}\right]-\left[E(e^{\mu+\sigma Z})\right]^2 \\ &= e^{2\mu+.5(2\sigma)^2}-\left[e^{\mu+.5\sigma^2}\right]^2 \\ &= e^{2\mu+.2\sigma^2}-e^{2\mu+\sigma^2} \end{aligned}$$

Substitution of the values for μ = 0.08895 and σ = 0.15 into the last equation implies that the standard deviation of R is 0.1667, which closely matches SFG's empirical estimate of 0.1664. Substitution of these values for μ = 0.08895 and σ = 0.15 into equations (4) now yields

$$U=e^{\sigma\sqrt{T}}=e^{0.15\sqrt{(1/12)}}=1.0442524$$

$$D=1/U=0.9576228$$

$$q=0.5\left[1+\frac{\mu}{\sigma}\sqrt{T}\right]=0.5\left[1+\frac{.08895}{0.15}\sqrt{(1/12)}\right]=0.585592$$

Using these values for *U* and *D*, the resulting possible outcomes over the course of one year are as shown in the central column of Table 2 below. Using the value for *q* determined above, the resulting probabilities for these possible outcomes are shown in the last column.[9] Deducting 1 from the outcomes to obtain the rate of return, the mean and standard deviation of this distribution are 0.1050 and 0.1628 respectively, which are within 1% and 2.5% of the empirical values underlying the calculations (0.1054 and 0.1667 respectively).

Table 2: Distribution of Returns from the Conventional Approach

No. Down	Outcome	Probability
0	$(1.044252)^{12} = 1.6814$	$(0.585592)^{12} = 0.00163$
1	$(1.044252)^{11}(0.95762) = 1.5419$	$12(0.58559)^{11}(0.41441) = 0.0138$
….	….	….
….	….	….
11	$(1.044252)(0.95762)^{11} = 0.6485$	$12(0.58559)(0.41441)^{11} = 0.00043$
12	$(0.95762)^{12} = 0.5947$	$(0.41441)^{12} = 0.00003$

[9] The coefficients on the probabilities in Table 2 are taken from Pascal's Triangle.

REFERENCES

Almeida, H., and Philippon, T., 2007, 'The Risk-Adjusted Cost of Financial Distress', *The Journal of Finance*, 62, pp. 2557-2586.

Amihud, Y., Mendelson, H., and Pedersen, L., 2005. 'Liquidity and Asset Prices', *Foundations and Trends in Finance,* 1, pp. 269-364.

Arrow, K., 1964. 'The Role of Securities in the Optimal Allocation of Risk Bearing', *Review of Economic Studies*, pp. 91-96.

Banz, R., and Miller, M., 1978. 'Prices for State-Contingent Claims: Some Estimates and Applications', *Journal of Business*, pp. 653-672.

Berk, J., and DeMarzo, P., 2014. *Corporate Finance,* 3rd edition, Pearson, Boston.

Black, F., 1976. 'The Pricing of Commodity Contracts', *Journal of Financial Economics,* 3, pp. 167-179.

Black, F., and Scholes, M., 1973. 'The Pricing of Options and Corporate Liabilities', *Journal of Political Economy*, 81, pp. 637-659.

Black, F., Derman, E., and Toy, W., 1990. ' A one Factor Model of Interest Rates and its Application to Treasury Bond Options', *Financial Analysts Journal*, 46, pp. 33-39.

Brealey, R., Myers, S., and Allen., F., 2011. *Principles of Corporate Finance,* 10th edition, McGraw-Hill Irwin, New York.

Breeden, M., and Litzenberger, R., 1978. 'Prices of State Contingent Claims Implicit in Option Prices', *Journal of Business,* pp. 621-651.

Brennan, M., and Schwartz, E., 1985. 'Evaluating Natural Resource Investments', *Journal of Business,* 58, pp. 135-157.

Copeland, T., Weston, J., and Shastri, K., 2005. *Financial Theory and Corporate Policy,* 4th edition, Pearson Addison Wesley, Boston.

Cox, J., Ross, S., and Rubinstein, M., 1979. 'Option Pricing: A Simplified Approach', *Journal of Financial Economics*, 7, pp. 229-263.

Damodaran, A., 2011. *Applied Corporate Finance,* 3rd edition, John Wiley & Sons, New Jersey.

Debreu, G., 1959. *The Theory of Value,* Wiley, New York.

Dick-Nielsen, J., Feldhutter, P., and Lando, D., 2012, 'Corporate Bond Liquidity before and after the Onset of the Subprime Crisis', *Journal of Financial Economics,* 103, pp. 471-492.

Dixit, A., and Pindyck, R., 1994. *Investment Under Uncertainty,* Princeton University Press, Princeton.

Grabbe, J., 1983. ‘The Pricing of Call and Put Options on Foreign Exchange’, *Journal of International Money and Finance*, 2, pp. 239-253.

Grinblatt, M., and Titman, S., 2002. *Financial Markets and Corporate Strategy*, 2nd edition, McGraw-Hill Irwin, Boston.

Hull, J., 1997. *Options, Futures and Other Derivatives*, 3rd edition, Prentice Hall, Englewood Cliffs.

Jarrow, R., and Turnbull, S., 1996. *Derivative Securities*, South Western College Publishing, Cincinnati.

McDonald, R., and Siegel, D., 1986. ‘The Value of Waiting to Invest’, *The Quarterly Journal of Economics*, 101, pp. 707-728.

Merton, R., 1973. ‘Theory of Rational Option Pricing’, *Bell Journal of Economics and Management Science*, 4, pp. 141-183.

PwC, 2012, *Transpower New Zealand Ltd*: *Leverage and the Cost of Capital*, report prepared for Transpower (www.comcom.govt.nz).

Ross, S., Westerfield, R., and Jaffe, J., 2013. *Corporate Finance*, 10th edition, McGraw-Hill Irwin, New York.

SFG, 2014. *Cost of Equity for the Goldfields Gas Pipeline*, report prepared for Goldfields Gas Transmission.

Trigeorgis, L., 1996. *Real Options*, MIT Press, Cambridge.

Welch, I., 2009. *Corporate Finance*, Prentice Hall, New York.