Estimating MRP for the ERA 2022 RoRI

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1 Introduction

1. AGIG has asked CEG to provide advice on the appropriate estimation and weighting of geometric average historical returns for the purpose of estimating investor's expected market risk premium (MRP). We have been asked to specifically review, to the extent relevant, the ERA's explanatory statement for the 2022 draft gas rate of return instrument and the Consumer Reference Group (CRG) 23 May 2022 submission to the ERA.[1]

1.1 Report structure and summary

2. All of our analysis is contained in section 2 of this report. Section 2.1 addresses issues surrounding the estimation of the geometric mean of historical excess returns. In our 4 May 2022 memo, we explained that the geometric mean MRP should be estimated as the difference between the geometric mean return on the market portfolio and the risk free rate over the same estimation window. In section 2 of this report we:

   - Update the geometric mean MRP estimates from our previous 4 May 2022 memo using 10-year rather than 5-year risk free rates/MRP.
   - Critique the ERA's reasons for not accepting our proposed method for estimating the geometric mean estimate of the MRP.

3. Section 2.2 addresses the question of what weight to give to the geometric mean.

   - Section 2.2.1 explains why, if constrained to rely on historical return series, the unconditional mean return on equity should give zero weight to the geometric mean and 100% weight to the arithmetic mean. In this section we explain why the ERA and, to a lesser degree the CRG, are mistaken to conclude that investment horizons above one year imply more weight should be given to geometric returns.
   - Section 2.2.2 explains that this is true irrespective of whether there is auto-correlation in the historical returns series. We also update our previous estimates of autocorrelation to include data from 2022.
   - Section 2.2.3 explains that, even if the ERA were not to accept the conclusions from section 2.2.1 and section 2.2.2, the maximum weight the ERA should give the geometric mean should be determined by the weighting scheme set out in the literature (e.g., by Indro and Lee (1997)) – where that weighting scheme seeks to estimate the expected return from a "invest and accumulate/reinvest returns" strategy over an investment horizon of not more than 10 years.

[1] CRG, Review of submissions on international equity beta and market risk premium issues raised in the ERA 2022 gas rate of return Focused Consultation Discussion Paper, 23 May 2022.

2 Estimating and weighting the geometric mean risk premium

4. This section addresses issues surrounding the estimation of, and weight given to, the geometric mean of historical excess returns.

2.1 Estimating the geometric mean excess realised equity premium

5. Our 4 May 2022 memo explained that the geometric mean excess return over the risk free rate should be estimated as the difference between the geometric mean return on the market portfolio and the geometric mean return from investing in the risk free rate over the same estimation window. By contrast, the ERA's method was to derive an annual estimate of the MRP (by subtracting an annual risk free rate from the annual market return) and then to compound the annual estimate of the MRP.

6. In Table 3-1 of that memo, we estimated that the correct estimation method raised the geometric mean by between 0.04% (2000-21 period) and 0.24% (1958-2021 period). Table 2-1 updates these estimates for the use of 10 year risk free rates and also for the addition of the 2022 calendar year.

Table 2-1: ERA vs correct method (10 year risk free rate)

	Estimation of MRP under geometric mean		
Time Period	Current ERA method	Correct method	Differences (Correct - ERA)
1958-2021	4.56%	4.81%	0.25%
1980-2021	4.77%	5.02%	0.25%
1988-2021	5.11%	5.25%	0.14%
2000-2021	5.30%	5.35%	0.05%
Average	4.94%	5.11%	0.17%

*Source: BHM dataset, CEG analysis. *Note that we can replicate the ERA estimates using a gamma of around 0.5 and an assumption that dividends paid each year is the difference between the stock price and accumulation indices. This may not be exactly how the ERA arrived at its estimates. However, for the purpose of our analysis it is the last column (the difference column) that is important and it will be trivially affected by any methodological differences.*

7. It can be seen that the differences between the ERA method and our method has remained in a similar range to that estimated in our 4 May 2022 memo which used a 5 year risk free rate. The average difference of 0.17% is 0.01% higher than the 0.16% average difference reported in our memo.

8. Our method is the correct measure of the geometric average historical excess return because it measures the relevant concept. Namely, the difference in annualised returns between investing in the market portfolio over the relevant period and the annualised returns from investing in the risk free rate over the relevant period.

9. The ERA's method, by contrast, does not measure the difference in returns between being invested in the market portfolio versus being invested in a risk free portfolio. Rather, the ERA measures a quite different concept. Namely, the ERA is estimating the absolute return on a strategy that involves borrowing (at the risk-free rate) 100% of the value of an investment in the market portfolio. The ERA's only engagement with our proposed approach was as follows.[2]

> *CEG's memorandum for AGIG provided an alternative geometric mean formula that it submitted should be considered instead of the ERA's current formula. The ERA is unclear why CEG believes that it is not possible to implement a trading strategy where an investor purchases the market portfolio and borrows the risk free asset to create a long-short portfolio that realises the market risk premium that is liquidated and reinvested across periods. Accordingly, the ERA will maintain the use of its existing formula.*

10. We do not consider that this is a well-reasoned rejection of our proposal.

11. First, such an investment strategy is impossible to implement. No rational lender would finance a risky investment by lending at the risk free rate. The ERA's investment strategy is not just a "risky" investment it is an extremely risky investment – being a 100% levered investment in equities.

12. Consider what would happen if, in the first year of the investment, the equity market fell. In that circumstance the investor would have less value invested in equities than the loan that they took out to fund the investment. The investor would be unable to pay back the lender. The lender, knowing this is a possibility, would never lend to the investor at the risk free rate.[3] Moreover, if they did, and the equity market fell below the value of the loan, the compounding of the investment would cease – as the lender would immediately foreclose on the loan.[4]

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2 ERA, Explanatory statement for the 2022 draft gas rate of return instrument, p. 128, paragraph 769.

3 One might be tempted to assume that the investor could still pursue this strategy if they posted collateral against the loan (e.g., gave the lender a mortgage over their house). If the collateral is large enough relative to the loan, then the lender may treat the loan as approaching a risk-free investment (putting aside expected value of transaction costs associated with actually foreclosing on the collateral). But in this case the investor is not, in reality, funding the investment solely with the loan. The investor is also funding it by putting their collateral at risk. The correct measure of the return on this investment is the return net of interest payments and the risk adjusted foregone return on the collateral.

4 Put mathematically, the return on the investment would be less negative 100% (the investment would have negative value) and estimating a geometric average in this context is impossible/nonsensical. This

13. In any event, the ERA is not estimating a risk premium; being the excess return above the return from investing in the risk free rate which is the definition of the market *risk premium* in the CAPM. Rather, the ERA is estimating the *absolute* return on a risky strategy funded 100% by borrowing at the risk free rate. These are not the same concepts and, hence, they do not give the same answers (at least not in all circumstances).

14. It is for these reasons that all academic and other credible estimates of the geometric mean risk premium from historical data use our proposed method. We are unaware of any expert party that uses the ERA's method. For example, Dimson, Marsh and Staunton for Credit Suisse provides the canonical estimates of geometric MRP estimates across international jurisdictions. The authors state: [5]

> *Investors expect a reward for exposure to risk. This is the risk premium, which we measure relative to the returns on Treasury bills and, if appropriate, government bonds. To do this, we estimate the geometric difference between the realized return on an asset and the risk-free rate of return that was available over the same period.*

15. Consistent with this, the authors present separately the geometric average returns for equities and bonds and report the equity risk premium as the simple difference of these two geometric averages. That is, Dimson, Marsh and Staunton for Credit Suisse adopt our method of estimating the geometric average.

16. Indro and Lee (1997) also compound returns (not risk premiums). (We shall discuss this paper in more detail below).[6] To the best of our knowledge the only other organisation that adopts the ERA's approach is the AER – although the AER does not explicitly rely on the geometric average MRP.

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problem exists because the ERA's hypothetical investment is made without the investor actually providing any of the savings to fund the investment (it is 100% debt financed). This means that any negative return turns the value of the investment negative.

5 For example, see Credit Suisse Global Investment Returns Yearbook, Summary edition 2021 available at: https://www.credit-suisse.com/media/assets/corporate/docs/about-us/research/publications/credit-suisse-global-investment-returns-yearbook-2021-summary-edition.pdf. See page 23.

6 Indro and Lee, *Biases in Arithmetic and Geometric Averages as Estimates of Long-Run Expected Returns and Risk Premia*, Financial Management, Vol. 26, No. 4 (Winter, 1997), pp. 81-90. In the opening paragraph of this paper, Indro and Lee make clear that the market equity risk premium to be estimated is the "*difference between the future expected return on the market index and the risk-free rate of interest*" over a "N month" period noting that the correct answer may depend on the value of N. Indro and Lee proceed to focus on correctly estimating the geometric return on the market over that N month period. Indro and Lee do not contemplate an approach that involves estimating the geometric average of excess returns.

17. We also note that the CRG, in reviewing our memo, supported our (and the relevant literature's) approach to estimating the geometric mean MRP.[7]

> *"As a final point, when calculating the geometric mean, the CEG proposition that it should be calculated as the difference in the geometric means for each of total market returns and the risk free rate seems reasonable based on the information provided."*

2.2 Weighting the geometric mean excess realised equity premium

18. Having established the correct measure of the historical realised compound equity market risk premium (geometric mean MRP), we turn to the question of what weight to give that measure as opposed to the arithmetic mean realised excess return? We show that:

- The correct objective for the ERA is to estimate the MRP used by investors discount returns on the market portfolio. For this objective, 100% weight should be given to the arithmetic mean.
- There are other concepts of "average returns" that are generated by specific investment strategies over longer investment horizons. The best estimate of the internal rate of return (IRR) for some of these investment strategies will involve taking a weighted average of the historical arithmetic and geometric means. For example:
  - The IRR on an investment strategy that involves maintaining a constant dollar value of exposure to the equity market ("invest and consume earrings") over some horizon will be best estimated by giving 100% weight to the arithmetic average return; and
  - The IRR on an investment strategy that involves reinvesting all earnings ("invest and accumulate") will be best estimated by giving the weights estimated by Indro and Lee (1979).
- While the IRR on the above (and other) investment strategies can be of interest, this is not relevant to the task before the ERA (i.e., estimating the MRP used to discount equity returns).
- If, nonetheless, the ERA, in our view incorrectly, determined that its task was to estimate the IRR on an "invest and accumulate" investment strategy then the ERA should use the weighting scheme from the literature (e.g., as estimated by Indro and Lee (1979)).

7 Consumer Reference Group, Review of submissions on international equity beta and market risk premium issues raised in the ERA 2022 gas rate of return Focused Consultation Discussion Paper, 23 May 2022.

19. As a general point, we note that the rationale for estimating the opportunity cost of capital from historical return series must be that we cannot accurately estimate a conditional discount rate – being the discount rate investors apply at a given time based on market conditions at that time. Rather, the rationale for using historical returns is that the best predictor of the future is what has occurred in the past. While we do not necessarily agree with this factual statement, for the purpose of this report we accept it as correct.

2.2.1 Arithmetic mean is the best estimate of investors' expected return which, in turn, is the discount rate relevant to the ERA

20. It can be shown, and is taught in finance textbooks, that the arithmetic mean historical return is the best estimate of the unconditional future expected return. For example, Brealey, Myers and Allen (2020) state that:[8]

> *Moral: If the cost of capital is estimated from historical returns or risk premiums, use arithmetic averages, not compound annual rates of return.*

21. The authors supply mathematical proof for this conclusion by way of example of an asset that has three equally probable possible returns each year being: -10%, +10% and +30%. The expected return is, by definition, the equal weighted average of these – being 10%.

22. Assuming efficient markets, this implies that investors must apply a 10% discount rate to that asset. Otherwise, they would bid up/down the price of the asset to change its expected return. Specifically, if investors discount rate were, say, 8.8% they would bid up the price of the asset which would decrease the expected return from 10% to 8.8%. However, if the assumption set out in the previous paragraph is accepted (i.e., three equally probable returns with an equal weighted average of 10%) then it must be the case that the opportunity cost of capital for that investment is 10%.

23. This result is, as the authors point out, incontrovertibly true. However, Brealey, Myers and Allen go on to point out that if that same distribution was true in every year in the past then we would observe historical series that gave:

- An arithmetic average return of 10%; and
- A geometric average of 8.8% (= $(0.90\times1.1\times1.3)^{1/3}$).

24. It follows that the arithmetic average of historical returns is the only correct estimate of the annual opportunity cost of capital and the geometric average will understate this value. If investors had a lower discount rate than 10% then returns would, by

8 Brealey, R., S. Myers and F. Allen, 2020, Principles of Corporate Finance, 13th edition, McGraw-Hill, p. 170.

definition, by lower than 10%. If we observe an arithmetic average return of 10% this must reflect the average discount rate that investors have applied in the past.

25. Berk and DeMarzo (2020), in a competing textbook make the same point:[9]

> *"…we should use the arithmetic average return when we are trying to estimate an investment's expected return over a future horizon based on its past performance."*

26. The ERA appears to acknowledge this fact at paragraphs 759 to 761 of the draft explanatory statement.

> *The ERA's regulatory task is to estimate an expected return on equity to determine revenue requirements under an access arrangement determination. This task is best met by utilising methods that align with this requirement. The arithmetic mean is the method that appears best suited to achieving this task.*

2.2.2 Serial correlation is irrelevant to the best unconditional estimate of MRP

27. The ERA follows the above correct statement with an incorrect statement at paragraph 762 to 763.

> *However, full reliance cannot be placed on the arithmetic mean in the presence of serial correlation and sampling error which would bias the arithmetic estimate.*

28. The ERA's statement that serial correlation causes the arithmetic average to be biased as an estimate of the unconditional return is not correct. The ERA's statement appears to be based on accepting submissions by the CRG that serial correlation in returns implied bias in the arithmetic average (see discussion in paragraph 684 of the draft explanatory statement). Specifically, in their 7 May 2022 submission, the CRG refers to negative correlation in stock returns as a reason to believe that "the arithmetic mean will overstate the best estimate of the market risk premium".

29. However, this statement is not correct. Even with autocorrelation the arithmetic mean remains the best estimate of the unconditional expected return. Negative autocorrelation simply means that the conditional estimate will be higher/lower if the previous period's return was lower/higher than average. Negative autocorrelation

9 Berk, J. and P. DeMarzo, 2020, Corporate Finance, 5th global edition, Pearson, p. 368.

does not mean that the arithmetic average will overstate the unconditional expected return.[10]

30. In its subsequent 23 May 2022 report, the CRG distances itself from its earlier claim. In that report the CRG notes the potential for autocorrelation in returns but does not repeat the claim that this leads to bias in the arithmetic mean. The CRG further refers to autocorrelation as "*a minor point*" and states *"But in any case, as explained above, the more important issue is establishing the relevant investment horizon."*

31. We agree with the CRG that this is a correct construction of the issues. We consider that the discussion of autocorrelation is a distraction from, and not relevant to, an analysis of whether any weight should be given to the geometric mean of historical returns. (Although, we note that we did test for autocorrelation and did not find strong evidence of its existence.)

32. We also agree with the CRG that putting autocorrelation aside leaves considerations of "the relevant investment horizon" as the only remaining grounds for justifying giving weight to the geometric average. Here, we refer back to the previous section 2.2.1 for why the best estimate of investors' expected return is independent of the "investment horizon".

2.2.3 What is the CRG/ERA "investment horizon" rationale for giving weight to the geometric mean?

33. At paragraph 768 the ERA states:

> *The ERA considers the recognition of compounding to be an additional reason to place some weight on geometric means. This is especially the case in setting efficient returns with the 10-year term for equity. If the appropriate perspective for the purposes of the CAPM is of a long-term investor, then the compounding of returns is a reasonable investor expectation and will be incorporated into the market risk premium estimate through using geometric averages.*

34. In this passage the ERA seems to be saying that:

- the expected IRR on an "invest and accumulate" strategy will be between the arithmetic and geometric means of historical returns; and
- therefore, some weight should be given to geometric mean returns

10 We do note that CRG's 7 May 2022 submission does supply a Damodaran reference that does make the claim that CRG repeats – but this claim by Damodaran is unfounded (and no explanation nor reference is provided by Damodaran).

35. The first dot point is correct, but the second is not for the same reason as set out in section 2.2.1 above.

36. However, because there is considerable confusion in the CRG and ERA discussion of this issue it is useful to go into some detail. Let us consider the example from Brealey, Myers and Allen (2020) where the market return has an equal probability of being -10%, +10% or +30% in any given year. The expected return is, by definition, the equal weighted average of these – being 10%.

37. This 10% expected return is the same as the IRR on an investment strategy that always keeps the same dollar value exposure to the market. This investment strategy involves consuming positive returns (or saving them in the risk free asset) and investing additional funds when the market returns are negative. [11]

38. But there are many other possible investment strategies available to the investor including an "invest and accumulate" strategy. The expected IRR on an "invest and accumulate" strategy will be 8.8%.

39. There is nothing special about the "invest and accumulate" expected IRR that says it must be the opportunity cost of capital. There are an infinite number of possible profiles over which an investor may accumulate and consume wealth – each of which would have a different IRR. Indeed, the "constant exposure to risk" strategy is arguably more consistent with a "normal" investment strategy over an investor's life cycle (i.e., over the saving and consuming parts of the investment strategy).

40. However, the key point is that the correct estimate of the opportunity cost of capital does not turn on identifying a particular "correct" approach for an investor smoothing their consumption. As explained by Brealey, Myers and Allen, it mathematically follows that if historical returns are assumed to represent the distribution of possible future returns, then the arithmetic averages, not compound annual rates of return, must reflect the opportunity cost of capital.

41. To make this plain, take our simple example where market returns are -10%, +10% or 30% with equal probability – such that the expected return is 10.0% but the expected IRR on an "invest and accumulate" strategy is 8.8%.

42. Now, let there also be a regulated firm with an equity beta of 1.0 (i.e., the same risk as the market).[12] If the regulator set the return on equity for that firm at 8.8% then

[11] That is, when the return on the asset was 30%/10% the investor would sell 30%/10% of their stake and invest that in the riskless asset. When the return was -10% the investor would top up their exposure by withdrawing from the riskless asset and investing in the risky asset.

[12] One can make this example very simple by assuming that the regulated business always moves perfectly with the market. Now imagine that the market return (and the return for the hypothetical beta=1.0 regulated business) was always either 20 percentage points above or below its mean return. This means that if, say, the arithmetic mean market return was 5.0% then the annual market return could be +25% or

that firm would be unable to attract any equity funding. Investors could simply invest in the market portfolio for the same risk and a higher expected return of 10.0%.

43. The regulator could try and reason with equity investors along the following lines:

> "*Look, I know you can invest in the market at the same risk for a higher expected return of 10.0%. But if you "invest and accumulate" in the market you will end up with 8.8% in the end – so why don't you just accept an 8.8% expected return in the regulated business?*"

44. To which the (hypothetical) investors would correctly respond:

> *"No. If you offer us an expected return of 8.8% and we "invest and accumulate" in the regulated business our IRR will be lower than 8.8%. This is for the same reason that the market IRR on the "invest and accumulate" strategy is lower than the expected return of 10% (i.e., actual returns on the investment in the regulated business will themselves vary around your 8.8% mean). If you want us to have the same "invest and accumulate" IRR as the market you need to give us the same an annual expected return of 10%.*
>
> *Anyway, who says that the IRR on an "invest and accumulate" strategy is a benchmark that we care about?"*

45. This example is useful because the assumption of beta equal to 1.0 removes differences in risk from the equation. The ERA (and the CRG) may be implicitly thinking that regulated businesses are less risky than the market and, therefore, need a lower annual expected return than the market and this is achieved by giving weight to the geometric average. This appears to be what the CRG is suggesting in the following passage from their 23 May submission:

-15% per annum and these had equal probability of occurring in any given year. But the fluctuation of returns around this mean causes the expected "invest and accumulate" annualised compound return over a longer horizon to be 3.1%(=$(1.25*0.85)^{0.5}$-1).

In this simple example it is obvious that the regulator must set the regulated businesses expected annual return to 5%. Only if the regulator does this will investors in the regulated business expect to earn a 3.1% then the "invest and accumulate" annualised compound return over a longer horizon.

By contrast, if the regulator sets the expected annual return at 3.1% then the regulated business's annual returns will fluctuate between +23.1% and -16.9% (i.e., the same ± 20% around the now lower annual mean). The expected "invest and accumulate" annualised compound return over a longer horizon will therefore be only 1.1% ($(1.231*0.831)^{0.5}$-1). That is, by setting an annual regulated return of 3.1% the regulator is ensuring that an investor in that business (adopting an "invest and accumulate" strategy) will achieve annualised compound return of less than 3.1%. This must be the case so long as the regulated business' actual returns fluctuate around the regulator's target return

To recap: Wright, Mason and Miles note that if returns have a log normal distribution, which is commonly assumed, then the relationship between the arithmetic return and the geometric return is as follows:32

> *arithmetic return = geometric return plus half the variance of the log normal return.*

Given the monopoly status of the regulated entities and the form of regulation that applies it is reasonable to recognise there is likely to be relatively low variance of returns to the regulated entities.

46. But, as correctly identified by the ERA,[13] this is a category error. Any such lower variance relative to the market is already picked up by the beta estimate. It is double counting to argue that this should also lead to a lower MRP estimate.

47. One further extreme example might help illustrate why the geometric mean cannot be correct – even if "the holding period" for an investment is infinite. Consider a very risky country where the value of investment in the market portfolio has a 50% probability of tripling in any given year and a 50% probability of being wiped out completely in any given year. That is, the investment either pays a return of 200% or -100%.

48. The expected return on investing in this market is 100% but the geometric average return is -100% (provided our time series has at least one year with a -100% return event – which is very likely given the annual probability of 50%). That is, if an investor places 100% of their wealth in the market and then leaves it plus all earnings to accumulate for long enough, the final return will be -100%.

49. But it would clearly be nonsensical to argue that this investment has an opportunity cost of capital of -100% (or an expected return of -100%). The opportunity cost of capital in this risky market is clearly +100% pa. Investing in this market overtime will deliver an average return of +100% pa. The reward for bearing that risk is, demonstrably, not negative 100% per annum (that would be a nonsensical result). On the contrary, investing in this market will generate high returns overtime provided that the investor does not invest and hold 100% of their wealth in the market. If an investor always has the same value invested the market (e.g., some specific fraction of their initial wealth) then their expected return will, on average, be a doubling of that value each year. For example, if an investor always has $100 invested in this market, then half of all years they will lose $100 and in the other half of all years they will gain $200. On average, they will be $100 pa better off.

50. We hope that this illustrates why it is wrong to believe that a long assumed "holding period" implies a high weight to the geometric average. Whether investors planned to stay invested in the market for 5 or 50 years they would still require the market

[13] ERA, Explanatory statement for the 2022 draft gas rate of return instrument, June 2022, p.127, para 767

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return for risk – which in this example is 100%. A simple (or 60/40 weighted) average of the historical arithmetic and geometric returns would results in an estimated market return of 0% (20%). If a regulator offered a 0% (20%) return for a beta=1 regulated asset in this hypothetical country then no investment would be forthcoming – because investors could expect to receive 100% from investing directly in the market portfolio (where the beta is also equal to 1.0).

51. The same logic applies generally to examples that do not rely on years where equity investment is wiped out completely. Imagine a market where the return could be -75% or -50% with 50% probability. Over a long period of time the geometric compound return from a "invest and accumulate" strategy will be -6.5% pa. But the expected annual return is clearly positive at 13% (which is also the expected IRR of a "invest a constant value in the market" strategy).

52. The above examples are, we hope, a helpful elaboration of the same concept that Dr Lally has previously communicated to the AER:

> *"The AER's belief that geometric averages are useful apparently arises from a belief that there is a compounding effect in their regulatory process (AER, 2012, Appendix A.2.1), and therefore the analysis of Blume (1974) and Jacquier et al (2003) applies. However, I do not think that there is any such compounding effect in regulatory situations and the absence of a compounding effect leads to a preference for the arithmetic mean over the geometric mean. If historical average returns are used, they should be arithmetic rather than geometric averages."*[14]

> *"The geometric mean fails this test whilst the arithmetic mean will satisfy it if annual returns are independent and drawn from the same distribution. So, if historical average returns are used, they should be arithmetic rather than geometric."*[15]

2.2.4 Any weight given should not exceed those specified by Indro and Lee (1997)

53. The analysis of the previous sections explains why the geometric historical average returns are not relevant to the ERA's task:

- Section 2.2.1 explains that the arithmetic mean must be the discount rate that investors use (to the extent that they rely on the past as the sole guide to future returns);

14 Lally, M., The cost of equity and the market risk premium, Victoria University of Wellington, 25 July 2012, pg 31-32

15 Ibid., pg 32

- Section 2.2.2 explains that this is true irrespective of whether there is serial correlation in returns (unless the ERA is attempting to set expected returns in the regulatory period conditional on realised returns immediately prior to that period);
- Section 2.2.3 explains that this is true even if investors have an "investment horizon" longer than one year.

54. It is correct that the compound annualised return from an "invest and accumulate" strategy over a given investment horizon will be lower than the arithmetic average return. However, for the above reasons, this fact is of no consequence to the ERA's objective – which is to set a target rate of return equal to the opportunity cost of capital (and around which will fluctuate the actual rate of return investors in regulated businesses receive).

55. If, however, the ERA does determine that the compound annualised return from an "invest and accumulate" strategy over a given investment horizon is relevant, then it should rely on the literature to arrive at the best estimate of that concept. Specifically, Indro and Lee (1997) [16] who arrive at essentially the same conclusion as other authors before and since (such as Blume (1974)[17] and Jacquier et al (2003)[18])

56. These authors do not dispute the finance theory as set out in Brealey, Myers and Allen (2020) and Berk and DeMarzo (2020). They address themselves to a separate question. Specifically, they ask what the best estimate is of the expected compound return from a "invest and accumulate" decision for N years given a data set of historical returns that covers T years. They answer this question by estimating the weight to be given to arithmetic and geometric average historical returns. Indro and Lee (1997) show that their weighting scheme is correct both when historical returns are iid and when there is negative autocorrelation.

57. The answer to this question does involve giving some weight to the geometric mean. However, it is important to note that this does not in any way contradict the conclusion from the previous section that the opportunity cost of capital should be

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16 Indro and Lee, *Biases in Arithmetic and Geometric Averages as Estimates of Long-Run Expected Returns and Risk Premia*, Financial Management, Vol. 26, No. 4 (Winter, 1997), pp. 81-90.

17 Blume, M.E., 1974, "Unbiased Estimates of Long-Run Expected Rates of Return," Journal of the American Statistical Association (September), 634-638.

18 Jacquier, Kane and Marcus, Geometric or Arithmetic Mean: A Reconsideration, Financial Analysts Journal, 2003, 59(6):46-53

based solely on the arithmetic average.[19] The question that Indro and Lee set themselves is:[20]

> *"In this paper, we examine the biases obtained by using the arithmetic or geometric sample averages of single-period returns to assess* ***the long-run expected rates of return*** *when there is both a time-varying and a stationary component in those returns." (*Emphasis added.*)*

58. Indro and Lee's (implicit) definition of "the long-run expected rates of return" is the return from a "invest and accumulate" strategy over a N year period. As explained in section 2.2.1, this is not the same as the expected return/opportunity cost of capital. Moreover, as explained in Section 2.2.3, in order to generate this "long run expected (compound) rate of return" the annually expected rate of return must (by definition) be higher (must be the arithmetic average return).

59. The formula proposed by Indro and Lee to best estimate the expected compound return on this strategy is given by:

Equation 1: Weighting methodology proposed by Indro and Lee (1997)

$$Weighted\ mean = \frac{T-N}{T-1}R_A + \frac{N-1}{T-1}R_G$$

Where: T = total number of time periods (observations), N = investment horizon, R_A = arithmetic mean, R_G = geometric mean

60. As illustrated in Equation 1, different time periods have a different weighting between arithmetic and geometric mean. As the investment horizon approaches a single year (N=1) then the weight to the arithmetic average approaches 100%. Similarly, as the investment horizon approaches the number of years of data (N=T) then the weight on the geometric mean approaches 100%.

61. The logic for this weighting scheme is entirely intuitive.

- The expected return over N years is the expected the geometric average of any N randomly selected observations;
- But the expected geometric return over a full estimation period of T years falls is below the expected return over N years when T>N;

[19] Indro and Lee do, on the first page of their paper, refer to Brealey and Myers (1991) who reach the same conclusion as Brealey, Myers and Allen (2020) that the arithmetic average should be used to estimate the cost of capital – assuming historical returns are iid. A non-careful reader might conclude that Indro and Lee are reaching a different conclusion to Brealey and Myers (1991) and Brealey, Myers and Allen (2020). However, this is not the case, they are simply answering a different question.

[20] Indro and Lee (1997) p. 81.

- Consequently, as T exceeds N less and less weight should be given to the geometric mean over T years.

62. In our review of the ERA draft explanatory statement, the only reason provided for giving any weight to the geometric mean is an implicit and explicit assumption that the ERA is seeking to estimate the same type of return as Indro and Lee (1997). [21] For the reasons set out in section 2.2.1, we consider that this is a mistake. However, if the ERA does not accept our advice on this, it should, at a minimum, adopt the weighting scheme proposed by Indro and Lee (1997) which is similar to that of other authors answering the same question.

63. The ERA referred to Indro and Lee's analysis on the biases in arithmetic and geometric averages for estimating the MRP in its draft gas rate of return instrument.[22] The ERA also noted that that this implied "an approach to adjust and minimise the bias of means" proposed by Indro and Lee amongst others.[23] Nonetheless, the ERA did not follow the literature and, instead, adopted a seemingly arbitrary 60/40 weight to arithmetic/geometric means.[24]

64. We set out below the effect of using the Indro and Lee weighting (with and without our correction to the method for estimating the geometric mean).

65. The four estimation periods used by the ERA have values for T of between 22 and 64 years. The question then becomes what value should be given to N? In our view, the only reasonable value for N is the maturity of the risk free rate. The maturity of the risk free rate is length of the implicit horizon for the risk free rate and, as a matter of internal consistency, the MRP should be estimated over the same horizon.[25]

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21 See paraphs: 747 – 753 and 768 of the ERA draft explanatory statement.

2222 Paragraph 748

23 Paragraph 752

24 Paragraph 773.

25 Note that 5-year risk free yields are generally below 10-year yields and 10 year yields are generally below 15 year yields etc. The ERA's historical return series are based on the gap between equity returns and 10-year risk free rates – implying a 10-year investment horizon. If a different investment horizon was selected when setting "N" then a different risk-free rate should be used in the historical return series.

Table 2-2: Weighting factors based on Indro and Lee's methodology with N=10

Time Period	T	Weights for arithmetic mean	Weights for geometric mean
1958-2021	64	86%	14%
1980-2021	42	78%	22%
1988-2021	34	73%	27%
2000-2021	22	57%	43%

Source: CEG analysis

66. The table below illustrates the final MRP under the ERA's weighting and Indro and Lee's weighting.

Table 2-3: ERA's weighting vs Indro and Lee's weighting

	MRP with current ERA geomean	MRP with correct geomean	Difference
ERA's weighting	6.02%	6.09%	0.07%
Indro and Lee's weighting	6.29%	6.33%	0.04%
Difference	0.27%	0.24%	

Source: CEG analysis

67. Adopting the Indro and Lee (1997) weightings and the CEG estimation method would increase the unrounded MRP from 6.02% to 6.33%.